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THE USE OF SUPERCONDUCTORS FOR STORAGE AND DISCHARGE OF ELECTRICAL ENERGY

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ABSTRACT

After recalling the elementary properties which led to superconductors being used for generating very strong magnetic fields, as well as the methods used to attain the required very low temperatures in practice, the author presents in its elementary form the principle of storage and liberation of energy. A more detailed analysis of the successive charging, trapping, storage and discharge operations shows up the different aspects of the problem and is followed by a review of the very small amount of research published up to now and also by a presentation of the original work which has been carried out in this field. It is already possible to use superconducting materials as they are (i.e. although their characteristics have been adjusted with a view only to obtain very strong magnetic fields). Much more will be obtained from superconductors when they have been correctly adapted to this new very promising application to the storage and discharge of electrical energy. N69-11685

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Chief Notations Used

Storage Circuit

L	self-induction coefficient of the storage circuit
r	electrical resistance (if any) of the storage circuit
$i(t)$	current in the storage circuit as a function of time
I_0	initial trapped current or maximum current to be charged
J_c	critical current density of the superconducting material
B_c	critical induction of the superconducting material
m	total mass of the superconducting material
W_0	initial trapped energy

Switch

$R_2(t)$	resistance of switch as a function of time
R_m	maximum value of $R_2(t)$
R_0	resistance (if any) of the "closed" switch
L_2	self-induction coefficient of the switch
$V_2(t)$	voltage at the switch terminals as a function of time
$i_2(t)$	current in the switch as a function of time
w_2	energy dissipated into the switch
K_2	this symbol designates the switch used to trap energy

Connections

R_3	resistance of the connections
L_3	self-induction coefficient of the connections
w or w_3	energy dissipated by Joule effect into the connections (cf. also "Temperature associated quantities")

Use

Z_u	impedance of use
R_u	purely dissipative impedance of use
L_u	purely coil impedance of use
C_u	purely capacitive impedance of use
$i_u(t)$	current in impedance of use as a function of time
I_m	maximum value of the discharge current

W_u energy transferred in use

K_3 this symbol designated (if appropriate) the switch used to connect the use

Generator

K_1 this symbol designated (if appropriate) the switch used to connect the generator

Temperature Associated Quantities

T temperature at a given point in the circuit

w energy transported by thermal conduction (cf. also "connections")

$\rho(T)$ electrical resistivity as a function of temperature

$\sigma(T)$ thermal conductivity as a function of temperature

Time Associated Quantities

τ_c duration of charge

τ_s duration of storage

τ_u operating time in cold of cryostat

τ period of oscillation (case of an oscillating discharge) or either duration of discharge or time constant of discharge (this is specified in text)

τ_2 standard time constant characteristic of the storage circuit

Efficiency

η efficiency in electrical energy

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J. Sole

1. The Superconductor /7*

When the temperature of a metal or metal compound is lowered, there generally may be observed a progressive decrease in its electrical resistivity [1]. Nevertheless, the electrical resistivity of some metals and compounds can abruptly become equal to zero [2] below a specific temperature [3] to [10]. This phenomenon was observed for the first time by the Dutch physicist Kamerlingh Onnes in 1911 with mercury. This temperature which is a function of the material, including its nature, structure and geometrical shape, is called "critical temperature."

In 1933 the German Meissner discovered experimentally that, in the case of the materials just mentioned, when resistivity was zero, the induction into the material remained not only constant but again was zero. This phenomenon, presently called the "Meissner effect," characterizes the superconductor. The latter thus not only behaves as a perfect conductor (zero resistivity) but also as a perfect diamagnetic material (zero induction).

When this superconductor is subjected to an increasing magnetic field, at a specific value of the field the material abruptly regains the properties of a normal conductor. It is no longer a superconductor. It is said to have made the transition from the superconducting state to the normal state. This value of the magnetic field which is a function of the material is called the "critical field." It could even be possible to restore the material to its normal state by passing through it a current greater than a certain value. This value of the current, still a function of the material, is called the "critical current."

The state in which the material is found is therefore a function of the /8
whole of the values of three quantities:

* Numbers in the margin indicate pagination in the foreign text.

T temperature of the material;

H magnetic field to which it is subjected;

I electrical current passing through it;

i.e. the position of a representative point of coordinates T, H, I, in a system of axes with three dimensions: temperature, magnetic field, current. When this point is located in a region sufficiently close to the origin of the coordinates (T=0, H=0, I=0), the material is in the superconducting state. When this point is located, on the contrary, in a region which is sufficiently far from it, the material is in the normal state.

Both these representative regions can be well separated diagrammatically:

either by a unique surface. This surface corresponds in this case to sets of "critical values" for T, H, I. The transition of the material is carried out completely with passage of the representative point across this surface and the material is called "superconductor of the first class." (This does not exclude the case, for example, where in a same massive specimen there can coexist superconducting zones and normal zones respectively corresponding to different sets of values of T, H and I. In this case it is said that the specimen is in the mixed state [3] [4]).

or by a whole region included between two extreme surfaces. The representative points located within this region correspond to a partial penetration of the magnetic flux into the material [11] [12] and [3] to [7]. The corresponding material is called "superconductor of the second class."

The behaviors described above are relative to a permanent mode, i.e. with constant values (or, at the limit, very slowly variable ones) of T, H and I. In transitory modes it is possible to observe an effect of "degradation" of superconducting properties [13] to [16].

Generally, during transition, the crystalline lattice of the superconducting /9 material undergoes no modification. On the other hand, the specific heat allows a discontinuity (although preserving the same order of magnitude) when a magnetic field is present. A latent heat of transition is the result in this case. The thermal conductivity is generally lower in the superconducting state than in the normal state for a pure metal whereas it is higher for some alloys [3] to [7] and [17].

At the present time, there are a limited number of elements, all metallic, known as superconductors [3] to [7]. There are also a great number of alloys and compounds [27] to [30] as well as some metalloids [27] known to be superconductors. The maximum critical temperatures of these superconductors were up until recently still less than 18° K. The discovery was made quite recently that there are materials which are still superconductors at 20° K [37]. Theoretically, superconductors having critical temperatures on the order of magnitude of the ambient temperature have been predicted [31] to [35], but such a possibility is still much debated [10].

Up until now, the most often used properties of superconductors have been:

- their practically zero electrical resistance;
- their high critical fields.

This has allowed production of windings with practically no losses (nevertheless, reference [36] on the degradation phenomenon should be consulted) and capable of creating intense permanent magnetic fields [11] and [18] to [26]. This is the use which has given superconductors of the second class their considerable popularity which they have gained in seven years. Nevertheless, as will be seen, the improvements which resulted from this led to production of materials having optimum characteristics differing from those required for storage and release of electrical energy.

2. How the Very Low Temperatures Required Are Reached

It was owing to the liquefaction of helium which he produced for the first /14
time in 1908, that Kamerlingh Onnes was enabled by chance to discover in 1911, the first superconductor.

At the present time, practically the same technique is still used to cool a superconductor. This cooling is still carried out beginning from natural liquid helium (helium 4) which boils at atmospheric pressure at the temperature of 4.2° K which the circuit to be cooled is directly immersed [38] to [44].

Helium has the lowest boiling point of all elements. Table 1 groups together those elements having the lowest boiling temperatures at atmospheric pressure.

TABLE 1

Element	Boiling point at standard pressure (degrees absolute) °K	Corresponding centesimal temperature °C	Specific gravity in liquid state g/liter	Latent heat of vaporization cal/g	Latent heat of vaporization cal/liter
Helium 3	3.2	- 269.9	≃ 50		< 100
Helium 4	4.2	- 268.9	125	4.9	610
Hydrogen	20.3	- 252.8	71	108	7,700
Deuterium	23.6	- 249.5	163	72.5	11,800
Tritium (radioactive)	25	- 248.1	255	55	14,000
Neon	27.1	- 246	1,207	20.7	24,700
Nitrogen	77.3	- 195.8	808	47.6	38,400

It is possible to reduce the boiling temperatures shown by decreasing the /15
equilibrium pressure of the gas which is found in contact with the liquid. Nevertheless, solidification of the liquid occurring as a result of relatively small temperature reductions places a limit on this.

In the case of helium 4 [45] to [47], there appears a new phenomenon in the vicinity of 2° K: superfluidity. The helium acquires a zero viscosity and is found to be literally sucked up by the pumps intended for lowering the pressure. Below 1.2° K it will then have to be replaced by its isotope helium 3 [40][48], although this latter is very rare and difficult to handle. It appears, furthermore, that presently there is no advantage in dropping too far in temperatures since the critical fields of the materials now used do not increase much when zero temperature is neared [3] to [7], whereas their thermal conductivities and calorific capacities decrease considerably [38] [39] [42] [44].

On the contrary, if it were desired to reach temperatures greater than those shown in Table 1, it would be possible to increase the pressure. On a practical basis, it is preferable to slightly heat up the circuit by supplying it with a suitable heat flux in such a manner as to keep it at an equilibrium temperature greater than that of the cooling fluid.

In reality, as shown in Table 1, helium is the only fluid which can be used on a practical basis to produce temperatures clearly below 18° K.

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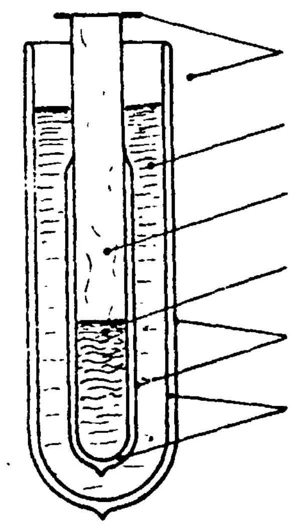

Figure 1.

The liquid helium is placed in a so-called "cryostat" container made up in reality by a system of Dewar vessels especially designed to reduce to the minimum the contribution of calories coming from the outside. Figure 1 shows a cryostat using a bath of liquid nitrogen which keeps an intermediate temperature of 77°K around the helium Dewar. The present-day tendency is to substitute for the liquid nitrogen bath a "superinsulating" thermal screen (thus simplifying handling and making the cryostat less bulky, lighter and more solid) or even to cause the still cold vapors of helium to circulate around the Dewar helium vessel.

Formerly, many cryostats were made of glass. However, this material is /17
quite brittle and is presently being replaced by stainless steel which is a

poor heat conductor at very low temperatures and usable as foils with very small thicknesses. In order to avoid appearance of currents induced into the mass itself of the cryostat during discharge of the superconducting circuits, it appeared appropriate to concentrate on the use of materials which are very poor conductors of the electrical current or have insulating properties [49]. On the other hand, the geometrical shape of the cryostats is adapted to those of the circuits to be cooled (for example, toric cryostats). In some cases, helium circulation devices were produced which allow the local cooling of just the required parts of the circuits [50] [51] [52].

Liquid helium is produced by liquefaction of the gas by means of refrigerating plants still called "liquifiers" [43]. It is then generally necessary to transport the liquid up to the cryostat of use. Instead of liquifiers, "refrigerators" can be used. The latter cause the cryogenic fluid to describe a directly closed circuit in the cryostat.

The useful refrigerant energy of the cryogenic fluid includes the latent heat of vaporization of the liquid (Table 1) plus the heat of reheating the gas to the temperature at which it is evacuated. For information purposes, Figure 2, taken from reference [50], provides directly for a consumption of 1 liter per hour of different cryogenic fluids, initially in the liquid state, and as a function of the evacuation temperature of the gas, the operating power collected expressed in watts.

This refrigerant energy is used:

1. to initially lower the temperature of the superconducting circuit to its temperature of use (this operation is carried out slowly in order to improve heat exchanges and to avoid subjecting the circuit to thermal shocks);
2. when this temperature is reached, to compensate for losses from the cryostat (since the Joule effect is zero in the superconductor). /18

Let us recall that these losses take place [53] by:

- convection of the gas contained in the cryostat (It should also be pointed out that there is convection of the gas which was able to enter, by diffusion through the walls, into the insulation chambers which should stay under vacuum.).

- thermal conduction (walls of the cryostat, supports for the circuit, various connections...).
- radiation (these losses are reduced by means of reflecting screens kept at intermediate temperatures).

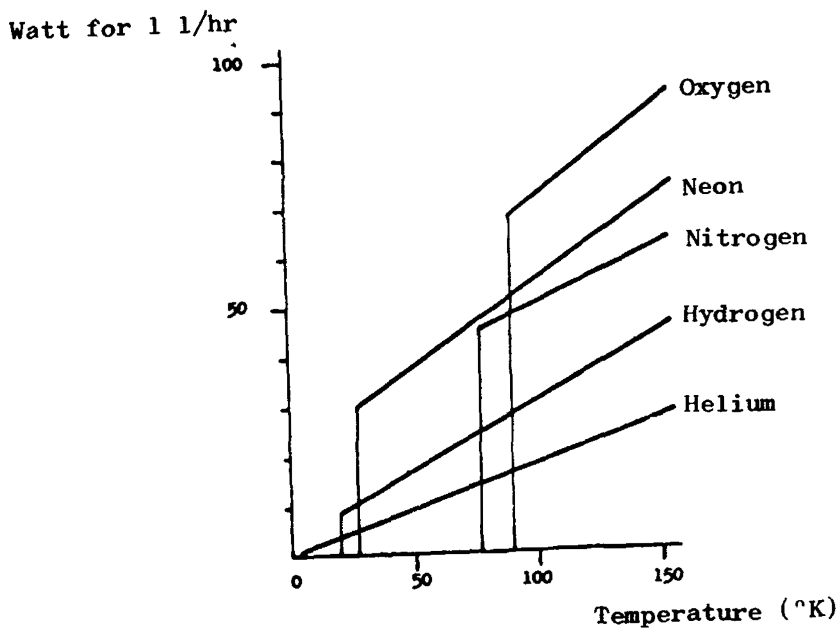

Figure 2.

If superconductors with higher critical temperatures were to appear, this would allow use of cooling fluids with higher boiling temperatures, thus making the cooling easier. Indeed, the last column of Table 1 as well as Figure 2 show that at atmospheric pressure, for example, the refrigerant power of the different cryogenic fluids is generally (except in the case of nitrogen) greater as their boiling temperature rises. On the other hand, since the deviation of temperature with the ambient temperature is smaller, the total losses from the cryostat are probably less. /19

Our present tendency is to concentrate our attention on a refrigerant device which can automatically ensure, and with every safety (by avoiding "thermal shocks"), the progressive refrigeration, keeping cold and rise in temperature in the superconductor circuit.

3. The Principle of Use of Superconductors for Storage and Discharge of Energy

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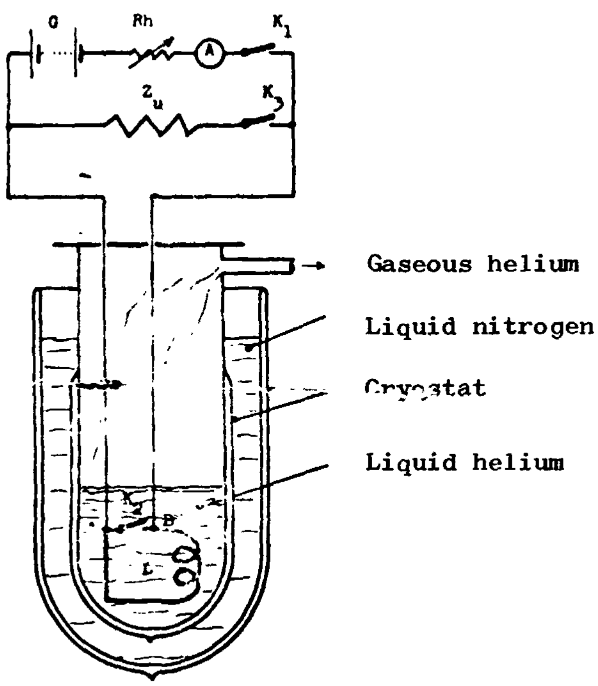

Figure 3.

Legend

G Current generator

Rh Rheostat

A Ammeter

Z_u Impedance of use

K_1, K_3 Switches

K_2 Supraconductor switch

A superconducting circuit with self induction coefficient L traveled by a current I stores, just as if it were made up from conventional conductors, an electromagnetic energy W which can be expressed [54] [55] (P. 6-1). /23

$$W = 1/2\ LI^2 \tag{1}$$

We shall see, on the other hand, in the following what new capabilities are contributed by the superconductors.

Figure 3 diagrams a circuit allowing, starting from an electrical generator G, energy to be stored in a coil L, then to release this energy in a use circuit with impedance Z_u. Only the circuit A, K_2, B, L A can be superconducting. The principle of operations is the following:

1) when coil L is superconducting and switches K_2 and K_3 open, switch K_1 is closed. By means of DC generator G and rheostat Rh, the current into coil L is progressively increased to reach a given value I_o. An energy $W_o = 1/2\ LI_o^2$ is then found to be stored in coil L.

2) current I_o is kept constant. The difference in potential between points A and B at the coil terminals can be expressed:

$$V = r\ I_o + L \frac{dI_o}{dt}\ . \tag{2}$$

Now: since the coil is superconducting, its resistance r is zero; since current I_o is kept constant the term $\frac{dI_o}{dt}$ is zero. The difference of potential V is therefore zero and points A and B have the same potential. It is then possible, without changing current I_o traveling through the coil, to connect A and B by closing switch k_2. /24

3) since this switch is superconducting its resistance is zero. If, in addition, its coeffecient of self induction is zero, it will be possible to progressively drop to zero the current throughputted by generator G without any difference in potential appearing between A and B. The generator G will then be switched off.

By writing that the difference in potential between points A and B at the coil terminals remains zero, it follows that:

$$r\ I_o + L \frac{dI_o}{dt} = 0 \tag{3}$$

and since the resistance r of the superconducting circuit is zero, it remains:

$$L \frac{dI_o}{dt} = 0 \tag{4}$$

whence it further follows that:

$$I_o = \text{constant}$$

i.e., the current I_o initially trapped in the coil has not changed. The initial energy W remains trapped in the superconducting circuit closed on itself.

4) in order to release this energy into impedance of use Z_u, this impedance will be connected by closing switch K_3, then the superconducting switch /25

K_2 will be opened. The trapped current will then flow into impedance Z_u whence the corresponding energy will be dissipated.

In the following, we are going to examine more in detail the main problems relating to these different operations.

4. The Charging of Energy

This charging can be carried out by means of an outside generator directly /27
supplying the DC current required as shown in Figure 3. This requires use of electrical connections.

It is also possible, as will be seen below, to carry out intermediate transformations of energy and even do away with any material electrical connection with the energy storage circuit.

4.1. The electrical connections:

They should at the same time be good conductors of the electrical current in such a way as to reduce the Joule effect during charging and poor conductors of heat in order to avoid contributing by thermal conduction too much calorific energy into the cryostat [56] [57].

Let us consider a connection with length l whose section s(x) is a function of abscissa x and whose ends are respectively at temperatures T_1 of the cryogenic device and T_2 of the outside environment (fig. 4).

Disregarding the quantities of heat exchanged by radiation and convection we are going to calculate successively:

1) the calorific energy released by the Joule effect w during duration τ_c of charging:

$$w = \int_0^{\tau_c} r_3 \, I^2(t) \, dt = \int_0^{\tau_c} \int_0^{l} \rho(T) \frac{dx}{s(x)} \, I^2(t) \, dt \qquad (5)$$

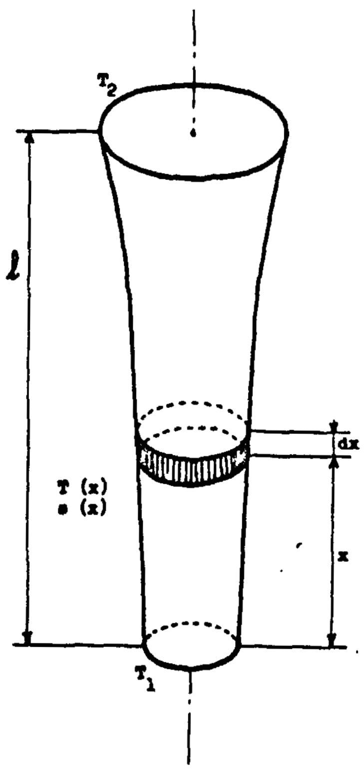

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Figure 4.

in which:

r_3 total resistance of the connection /29

$\rho(T)$ electrical resistivity of the material making up the connection (as a function of temperature T)

$I(t)$ electrical current.

2) <u>the calorific energy contributed by thermal conduction w'</u> during total duration τ_u of the operation:

$$w' = \int_0^{\tau_u} \sigma(T) \; s(x) \frac{\partial T}{\partial x} \; dt \tag{6}$$

in which σ (T) is the thermal conductivity of the material forming the connection, as a function of temperature.

3) <u>total calorific energy W</u>:

$$W = w + w' = \int_0^{\tau_u} \int_0^{l} \rho(T) \frac{dx}{s(x)} I^2(t)\, dt + \int_0^{\tau_u} \sigma(T) \; s(x) \frac{\partial T}{\partial x} \, dt \tag{7}$$

It should be made minimum. Now, in expressions (5) and (6):

a) s(x) becomes a factor <u>in reverse</u>;

b) length l of the connection becomes a factor in reverse (since the higher l becomes, the higher the numerical value of w and, T_1 and T_2 being given, the lower the values of $\frac{\partial T}{\partial x}$ in w');

c) in the case of a metal at a given temperature $\rho(T)$ and $\sigma(T)$ vary in reverse (in reality these two quantities are connected by the Wiedeman-Franz rule:

$$\rho(T)\ \sigma(T) = 2\ \left(\frac{k}{e}\right)^2 T$$

in which k is the Boltzman constant and e the electron charge).

At the most it will therefore no longer be possible to reduce w without increasing w' and vice versa. The optimization of the connections corresponds to the best compromise in the selection of the nature of the material, its shape and size, its arrangement and corresponding temperature gradient. Different materials can furthermore be simultaneously used in such a way as to make minimum the value of W given by expression (7). Indeed, in order to connect the temperature to abscissa x and time t, there should be taken into account those energies exchanged by radiation and convection. Optimization is therefore multiparametric and difficult to determine. /30

At any rate, the function $I^2(t)$ which becomes a factor in the expression of w assumes values which become higher proportionally as the value of the maximum current I_o to be established reaches higher figures. In order to avoid an excessive increase of w, it appeared appropriate to use values which were proportionally higher for the section s(x) which became a factor as a denominator. This leads to an increase of w' as well as an increase of W.

The result is that the connections turn out to be more difficult to use when the maximum intensity I_o of the current to be transported becomes higher. With the present materials, the "optimization" of relation (7) already provides considerable calorific energies as soon as the currents reach several thousands of amperes.

When, on the contrary, the connections are not perfectly "optimized," their capabilities become very quickly reduced. We are going to see it using a simple example in which section s of the connection is constant with a length l' subjected to a temperature difference $T_2' - T_1'$.

It is assumed that temperature difference $T_2' - T_1'$ is small enough so that there could be considered in this interval mean values ρm and σm for ρ and σ.

In this case, expression (7) becomes: /31

$$W = w + w' = \rho_m \frac{l'}{s} \int_0^{\tau} I^2(t)\,dt + \sigma_m \frac{s}{l'} (T_2' - T_1')\,\tau_u \tag{8}$$

Its first derivative $dW/d(l'/s)$ is zero for

$$\frac{l'}{s} = \sqrt{\frac{\sigma_m}{\rho_m} (T_2' - T_1') \frac{\tau_u}{\int_0^{\tau} I^2(t)\,dt}} \tag{9}$$

Its sec ond derivative $\frac{d^2W}{d(\frac{l'}{s})^2}$ is in this case, positive. Consequently, this value of $\frac{l'}{s}$ corresponds closely to a minimum for W. (Furthermore, it could be seen with (8) that for $\frac{l'}{s} = 0$ it follows that $W = \infty$ and for $\frac{l'}{s} = \infty$ it follows that $W = \infty$. Between these two extreme values of $\frac{l'}{s}$, the value of W, therefore, passes through a minimum).

W is then equal to:

$$W_{min} = 2\sqrt{\rho_m \sigma_m (T_2' - T_1')\,\tau_u \int_0^{\tau} I^2(t)\,dt} \tag{10}$$

and, in this case, it follows that $w = w'$.

When the charging of energy is carried out with $\frac{dI}{dt}$ constant and calling I_o the maximum current to be charged, it follows that:

$$\frac{dI}{dt} = \frac{I}{t} = \frac{I_o}{\tau_o} \tag{11}$$

whence:

$$W_{min.} = \frac{2}{\sqrt{3}} I_o \sqrt{\rho_m \sigma_m (T_2' - T_1')\,\tau_u \tau_o} \tag{12}$$

Now, $T_2' - T_1'$ as well at τ_u and τ_c are set and once the best material has been selected, i.e. the one providing the minimum value with the product $\rho m\, \sigma m$, it can be seen in this example that the minimum of calorific energy W_{min} increases proportionally to the current to be charged I_o.

In reality, the values of section s(x) will be "optimized" as a function of the abscissa (relation (7)). In some cases, different materials will even be used. In all cases, the refrigerant energy still contained in the helium vapor will be recovered in order to cool off the connections before its removal from the cryostat [58]. /32

In spite of all these improvements, as soon as it concerns the direct introduction into the cryostat of high currents, a limitation is quickly reached of the quantity of calorific energy contributed. It appears more appropriate in this case to seek other solutions.

4.2. Dynamos and generators:

Generally concerned here are "cryogenic" versions or "superconducting" versions of conventional machines known under the same name (cf. P. 4.5).

These dynamos and generators [59] to [62], whose induced circuit is generally superconducting, allow transformation of a rotational mechanical energy into electrical energy. It is possible to produce in this way intense currents [60] directly into the superconducting circuit.

The mechanical energy can be introduced into the cryostat in various ways, for example:

- using a mechanical shaft. The latter can transmit the required power with a high torque and low rotational velocity, or with a low torque and a high rotational velocity. This last case allows using a shaft with smaller cross section but can require, on the other hand, in the low temperature region, a velocity reduction gear [73] avoiding in this way exposing superconducting windings to too high-speed variable modes of operation [13] or putting helium in motion at too great a velocity.

- or, using an electromagnetic coupling between the cold region of the cryostat and the outside [73]. /33

- or, using an "electrical shaft," simple electrical line allowing introduction of the required power with a lower current and higher voltage and supplying a motor, coupled mechanically in the low temperature region to the superconducting generator [73].

4.3. Flux pumps:

A typical example of flux pump is provided in reference [63]. In reality, in spite of appearances, this work only concerns constructional and operational differences relating to dynamos and generators which we have just spoken of above. Furthermore, some flux pumps are just like others. In all cases, the basic principle used is the same. The term "flux pump" only results from a way of looking at an unaccustomed situation before using superconductors (P. 4.5).

Many studies concerning these devices will be found in technical literature [59] [60] and [62] to [66]. The latter allow carrying out consecutive cycles. Each cycle allows introduction of a magnetic flux (i.e. a current) into the closed superconducting circuit. This closed circuit is never closed during operation. This allows preservation of the flux (hence the current) which has been trapped during previous cycles. These pumps, furthermore, can operate equally well introducing, on a progressive basis, a magnetic flux into a superconducting circuit as removing a magnetic flux from the latter.

It is possible to characterize: /34

- mechanical flux pumps using a magnet (permanent magnet, conventional electromagnet, superconducting "magnet"...) mechanically actuated. This magnet, when it is not superconducting, can also be placed in a non-cooled zone [67] outside the cryostat which removes any mechanical connection between the hot parts and the cold parts.

- static flux pumps in which the mobile field (sliding field, rotating field...) instead of being created by a mechanical movement is produced electrically by means of variable currents travelling through the stationary windings (or inductors) suitably arranged in the space. The maximum intensity of the inducting current can be reduced with a corresponding rise in voltage. It is possible, as above, to arrange the inductors outside of the cryostat in order to remove all material connection with the cold parts. It is possible, on the contrary, to also use superconducting inductors.

In many designs, the configuration used does not allow production of a good magnetic coupling between the inducting circuit and the induced circuit, necessarily leading to large losses of energy. However, one of the chief causes of losses is common to the flux pumps and dynamos and generators just discussed (P. 4.2) when they are used as described in technical literature to charge a coil superconducting use circuit. This cause of losses has a fundamental origin. It comes from the degradation of energy occurring at the time of its transfer from the coil inducting circuit into the coil use circuit. The results of the calculation carried out in P 7.3.2 can more particularly be applied to the flux pumps which we have just described. For this reason, /35 these devices, such as they are presently used, will not be suitable for charging large amounts of energy into coil use circuits, since the energy released in the form of heat is in this case too great in absolute value. We shall see in P 4.4 a way of avoiding this difficulty.

4.4. The use of static transformers:

It is possible to introduce energy into the cryostat by means of only slightly intense currents. They are transformed into more intense currents inside the cryostat by means of transformers. Such transformers have already been the subject of experimentation [68]. It is then advantageous to utilize transformers whose secondary at least is superconducting and which have no magnetic circuit. Indeed, the presently known magnetic materials, owing to their saturation with relatively low inductions, would not allow use of the very high critical field of the superconductors [70]. By removing the magnetic circuit, the losses of which it is probably the cause is removed and the circuit is unburdened. On the other hand, precautions have to be taken to produce a good coupling between primary and secondary [70] [71].

Nevertheless, when the use coil is large, such a transformer could not allow introduction of very high currents [68] and [70]. In this case, the transformer is caused to operate like a flux pump by carrying out consecutive cycles by means of switches [61] and [69]. The degradation of energy is removed, as discussed in P. 4.3, by carrying out exclusively switchings allowing reversible energy transfers [70] [70a] and [72] (such a process can easily be extended to some flux pumps, e.g., the mechanical piston pump already mentioned in reference [63] and which is also described in reference [5].) In order to avoid any loss of energy, switches are additionally used which are completely superconducting when they are closed.

In the case of the static transformer, the losses are then only caused by /36
the power supply connections. If a suitable transformation were used, it would be possible to reduce these losses to a minimum by using a sufficient low power supply current. To this should be added the energy required for control of the switches.

4.5. Comment:

In reality, the technical literature appears to show some degree of confusion between dynamos and superconducting generators, flux pumps, devices using static transformers, rectifiers or switches. In the end, all these devices are converters of energy which allow introduction of a magnetic flux or electrical current (which amounts to the same thing), i.e. an electrical energy into a superconducting circuit. It would certainly be preferable to discriminate between the devices supplying this electrical energy beginning from a mechanical en-.gy and those which supply it beginning from an electrical energy.

5. The Trapping of Energy

Depending on the charging method used, various conditions were found to be true. We are going to examine the various cases.

5.1. Charging was carried out by means of an outside generator:

When the superconducting circuit is found to be open, it is first of all necessary to close it. For this purpose, a "switch" is used which is installed at its terminals. This switch should be superconducting when it is closed if dissipation of energy after its closure is to be avoided. Likewise, its junctions (if any) to the superconducting circuit should be superconducting.

Once the switch has been closed, it is possible to return to zero the current supplied by the generator (P. 3.3). We are going to see that this current should nevertheless be very progressively reduced for if, at the instant at which the switch is closed, the current passing through it is zero, (the difference in potential at its terminals being zero), this current rises progressively up to the value of the stored current to the extent which the current supplied by the generator is returned to zero.

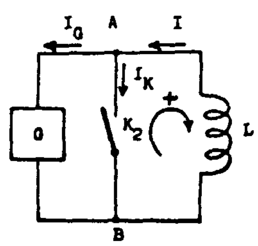


Figure 5.

Legend

I_G = current supplied by the generator

I_K = current passing through switch K_2

I = current passing through the storage coil

Indeed, let us consider the diagram shown in Figure 5 (corresponding to the circuit of Figure 3): applying the Kirchoff equations to this circuit, we have /40

node A: $I = I_G + I_K$

(the positive directions are those shown in Figure 5) and $\frac{LdI}{dt} = 0$ (14)

(for the superconducting K_2 switch does not show either coil or resistance between A and B)

From (14) it follows that:

$$I = \text{const.} = I_o \tag{15}$$

in which I_o is the value of the current initially trapped in the coil during closure of K_2.

From (13) it follows that:

$$I_o = I_G + I_K. \tag{16}$$

With the conventions of signs of Figure 5, it is true that:

at the initial instant $I_G = I_o$
whence it follows that $I_K = 0$
at the final instant $I_G = 0$
whence it follows that $I_K = I_o$. /41

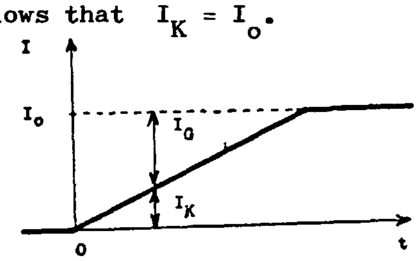


Figure 6.

Current I_K in the switch therefore passes from the value zero to the value I_o, this requiring the slow lowering of the current supplied by the generator in such a manner as to avoid values of $\frac{dI_K}{dt}$ which would be too high in the superconductor making up the switch. For example, if operations were carried out with a constant $\frac{dI_K}{dt}$, a diagram of the currents as a function of time t would be produced such as the one shown in Figure 6. Indeed, the optimum rule of variation will have to be determined in each case.

When the switch has a not inconsiderable self induction coefficient, the Kirchoff equation (14) relative to the link formed by this switch and the storage coil becomes:

$$L \frac{dI}{dt} + L_2 \frac{dI_K}{dt} = 0 \tag{17}$$

i.e. by removing I_K by means of (13):

$$\frac{dI}{dt} = \frac{L_2}{L + L_2} \frac{dI_0}{dt} \tag{18}$$

or again

$$dI = \frac{L_2}{L + L_2} dI_0 \tag{19}$$

Therefore, during the drop in current supplied by the generator, the intensity of the current in the storage coil is going to be reduced proportionally to the ratio $\frac{L2}{L+L_2}$ In the case of $L_2 = 0$, it is found that the current in the storage coil remains constant. /42

It is nevertheless possible to use a switch which is not superconducting. In this case, in order for the corresponding dissipation not to be too troublesome, it is necessary for the time constant $\frac{L}{R_0}$, determined by storage coil L and the resistance $R_2 = R_o$ of the closed switch, to be large enough with respect to the duration of energy storage (P. 7.5).

5.2. Charging was carried out by means of an inside generator:

Whether or not this generator is a dynamo, generator, flux pump or a static transformer:

- if its secondary circuit is superconducting, the energy storage circuit is always found to be closed by this superconducting circuit;

- if its secondary circuit is not superconducting, a return is made as before to the introduction of a dissipative time constant into the storage circuit. A superconducting switch can always be added which, when the generator halts, recloses the energy storage coil on itself.

6. The Storage of Energy /43

When the circuit is completely superconducting, it is possible in this way to preserve the trapped energy on an infinite basis. Indeed, when the superconductor is a second class material and when it is used in the vicinity of its critical characteristics, it is possible to observe [36] a slight decrease in the trapped current as a function of time.

6.1. Expression of stored energy:

The stored energy may be stated:

$$W = \iiint_v \frac{B^2}{2\mu_r \mu_0} dv \quad \text{which can be written again } W = \frac{1}{2} L I^2 \quad (20)$$

dv is the component of relative permeability current volume μ_r where induction B prevails as created by the superconducting circuit, v represents all the space encompassed by the magnetic induction, μ_o the absolute permeability of the vacuum.

The second expression has already been given by formula (1). It amounts to disregarding the energy trapped in the superconductor itself. This trapped energy is always slight in the case of energy storage circuits in which the volume occupied by the superconductor is small in contrast to the volume of space encompassed by magnetic induction. (If superconductors of the first class (P.1) were used, this energy trapped in the material with the superconducting state would be strictly zero since it was seen in P. 1 that induction into the material was zero. In reality, the materials used will be superconductors of the second class owing to their much higher critical fields. Still the energy trapped in the material will remain very slight in our applications.)

On the other hand, stored energy W can be included between zero and a very high upper boundary which is difficult to calculate at the present time.

6.2. Selection of L and I: /44

Since energy W is set, we are going to see that it is possible to arbitrarily select either the value of coil L or that of current I. Once a selection has been made of the geometrical shape of the storage circuit, its outside dimensions are then only a function of the stored energy.

Indeed, once a circuit has been assumed as produced, the result is that there is in each basic domain of space M (Figure 7) a specific value of the induction vector $\vec{B}$, i.e. according to relation (20), a specific value of the total stored energy. Let us consider any closed circumference (C) surrounding the circuit. According to the Maxwell equations governing the laws of electromagnetism, the circulation of vector $\vec{B}$ along circumference (C) is equal to the flux of current I across any surface (S) being dependent on this circumference. This flux is the same if, without modifying the spatial distribution of the current, it is assumed that the total current is broken up in N streams with respective intensities $\frac{I}{N}$ independently of the fact that these streams flow through or not a same conductor. Induction $\vec{B}$ at any point whatsoever of space does not change and the same is true, according to (20), of the stored energy.

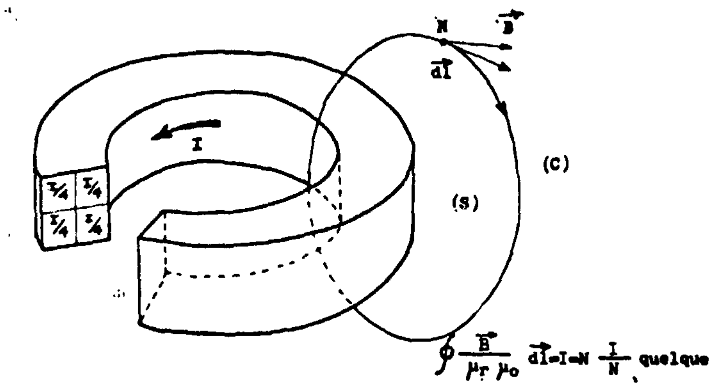


Figure 7.

irrespective of N (with N = 4 on the figure).
Therefore, $\vec{B}$ is independent of N.

The result is that if a coil circuit L traveled through by a current I stores an energy whose value is provided by the second expression (20), a circuit formed by N times more of windings, each one traveled through by a current N times weaker but having the same geometrical shape and the same outside dimensions, will store the same energy. It will therefore follow by designating by $\mathcal{L}$ the coil of this new circuit: /45

$$W = \frac{1}{2} L I^2 = \frac{1}{2} \mathcal{L} \left(\frac{I}{N}\right)^2 \tag{21}$$

i.e.:

$$\mathcal{L} = N^2 L \tag{22}$$

Consequently, since an energy W is set and the form of the storage circuit selected, neither the outside dimensions of this circuit nor the induction produced are a function of the value of the coil or storage current that has been selected. A choice can therefore arbitrarily be made, either L or I.

6.3. Stability of circuit:

A circuit storing large energy densities would be as dangerous as it would be less stable.

All the methods currently used to stabilize windings of magnetic fields, consisting in short circuiting turns or sections of windings, are to be eliminated (P. 7.6). What remains is the means of producing the storage coil directly beginning from "stabilized" materials such as those which are presently available on the market (superconducting cables or strips which are either sheathed or backed up with conductors). On the other hand, the switch (P. 7.5) cannot be produced with such a material. Nevertheless, it is always possible, during the duration of trapping, to influence its temperature (P. 7.5.1) [55] or even shunt it, for example by means of a superconducting mechanical switch (P. 7.5.2) which can itself be stabilized.

Nevertheless, the results of our works show (P. 7.5.1 - mixed switch) /46 that there are advantages in making the storage circuit itself operate as its own switch. In this case, the current methods of thermal stabilization are still usable to the extent in which neither the winding nor superconductor forming it are shunted electrically. In this case, materials which are good conductors of heat at the temperature contemplated are used as electrical insulators. On the other hand, the current methods of electrical stabilization are no longer usable (P. 7.6). Production of other types of windings is then suggested.

In the conventional windings the turns are mutually connected in series, i.e. the current passes successively through them one after the other. The probability that this current will encounter a defect along its path becomes greater as the number of turns increases. On the other hand, according to P 6.2, in the case of a given size and stored energy, the useful section of the superconductor formed by each turn becomes smaller as the number of turns increases. Now, when a defect appears in the superconductor, the section used for the passage of the current will be smaller as the defect increases in size. Under these conditions, there can locally result a transition of the superconductor in its whole useful section leading in the corresponding region to the release of <u>all</u> the energy stored in the circuit.

One solution which we have conceived [74] consists then, in the case of the same stored energy (P. 6.2), in reducing the number of turns and consequently increasing the section of the superconductor forming them (i.e. increasing the value of the current used). The optimum is reached at the limit when the winding is made up by one turn. It is then possible to produce it by means of a massive superconductor forming one closed winding. Indeed, in order to better "freeze" in space and time the distribution of the current in the material, and avoid in this way instabilities which could appear in this distribution the single massive turn, closed on itself, is subdivided into a great number of turns each one closed on itself with each one conductively mutually independent. The circuit is then formed by closed turns traveled by currents in parallel. It can even be formed by foils or layers each one closed on itself [74a] or even by rings also with each one closed on itself. Each one of the layers in question can furthermore be formed by a great number of conductively independent rings. Such circuits can be produced with materials laid down in successive layers, each layer being formed by independent rings, and then each layer closed on itself. It is possible in a more general way to use a massive material having anisotropies in its superconducting properties in such a manner that the lines of current cannot be shifted perpendicularly to the direction of propagation of the current. /47

These "massive" circuits then allow, provided their size is suitable, the superconducting material used to preserve properties close to those which it reveals in the form of "short samples. The result is that there is a much better utilization of the material. In addition, since the section used for passage of current is large in this case, this current is no longer interrupted by the presence or appearance of a local defect in the superconductor as has occurred with disastrous consequences in the case of windings formed beginning from a cable or, so much the worse, beginning from a simple wire.

Such methods also allow production in the storage circuit of more uniform distributions of the magnetic field and use of different successive materials according to the magnetic conditions in which they are placed owing to their respective geometrical positions in the circuit. In a conventional winding in which it is the same wire which successively forms the different turns and different layers, there are not such great capabilities. There is found, for example, in technical literature a description of windings, intended for production of intense magnetic fields, which have been prepared by concentrically arranging several independent windings produced with wires or cables of different natures [75]. This is a first approximation.

All this leads to storage of energy provided under a high current in a low value coil (P. 6.2). This neither stops charging energy into the circuit (P. 4.2, 4.3, 4.4)[74], one part at least of the storage circuit forming in this case the armature of the energy charging device (for example [74]), nor releases the energy under a current different from the one under which it is stored, nor correctly adapts the impedance of use (P. 7.2). /48

6.4. Comparison with standard conductors:

In conventional windings intended to operate in a permanent mode at ambient temperature, a limit is set by the Joule effect at current densities on the order of 100 to 500 A/cm^2. For example, a copper winding [76] producing an induction of 88 kG in a useful volume of 0.2 liter dissipates by Joule effect and per second an energy of 1.5 megajoule (whereas the magnetic energy stored in this useful volume is, under these conditions, less than 10 kilojoules). Such a winding requires a cooling water system flowing at a rate of 5,000 liters/minute.

It could be sought to store higher energies without increasing the Joule effect. For this reason, it would be necessary to lower the temperature of the conductors in such a manner as to reduce their resistivity [1]. Under these conditions, even if the latter were divided by 1,000, the Joule effect, proportional to the square of the current, would only allow increasing the current by a factor $\sqrt{1000=33}$. But then this same dissipated calorific power would have to be removed at a very low temperature which would not be practically possible in this example.

On the other hand, it is now possible to admit into superconductors of the second class current densities which can exceed 10^5 to 10^7 A/cm^2 [77][78][79][10] at 4.2°K in the presence of high magnetic fields. Under these conditions, although the critical characteristics of the material are not exceeded, a circuit /49
of given dimensions produced with presently available superconductors could store up to $\frac{(10^7)^2}{(10^2)^2}\ 10^{10}$ times more energy than the same conventional circuit. The cooling down of the superconducting circuit is relatively easy considering its small relative dimensions and absence of Joule effect, "flux jumps" [80][81][82] and "creep" [36].

6.5. Comparison with capacitors:

The best available capacitors allow storage in their dielectric of energy densities on the order of several tens to several hundreds of joules per liter [54][83] whereas relation (20) shows that the superconductors already allow, for example at 100 kG, storage of energies of 40 kilojoules per liter in the dielectric. This latter costs nothing and is formed by the ambient environment since with these inductions any magnetic core should be removed owing to its saturation.

The energy stored under optimum conditions by a capacitor is proportional to the total volume of the dielectric, meaning that the volume and, consequently, the weight and cost of one set of capacitors are perceptibly proportional to the stored energy. With superconductors, the situation is quite otherwise. With constant magnetic induction on the superconductor, the stored energy increases more quickly than the volume or weight of material used (P. 6.6). The result is that the cost of the stored joule decreases as the total stored energy increases [54]. This fact makes superconductors even more advantageous when the energy to be stored is higher.

We shall also see, at the time of discharge with a dissipative impedance (P. 7.3.1), another interesting property of superconducting circuits: they behave like "current generators" whereas the capacitors behave like "voltage generators." In the case of superconducting circuits, it is the initial current which is set before discharge whereas, in the case of capacitors, it is the voltage.

We shall again observe that considerable progress connected with the super- /50
conducting material (P. 10) itself can be expected.

6.6. "Optimization" of the storage circuit:

The circuit should be optimized as a function of the goal to be attained. Since the energy to be stored is given, this goal can be, for example, the producing of minimum cost, minimum weight, or even minimum volume, etc. The optimum storage circuit under these conditions is not the one allowing the reaching of maximum induction in the dielectric. The problem is totally different from the one which consists in seeking, with the given material, to create the maximum magnetic field.

Since superconductors are still rather difficult to handle, they do, on the other hand, allow reaching energy densities much greater than those allowed by capacitors. The optimization problem which often arises at the present time is to arrive at a minimum cost [54]. This appreciably amounts to wishing to store the given energy by using the minimum quantity of superconductors.

When formula (20) is examined too quickly, it could be thought that this condition will be satisfied if the maximum values of B are produced. In reality, this is not the case. It is the integral of B^2, stretching over all the space encompassed by the magnetic induction which should be made maximum. The calculation is carried out by seeking the maximum of this integral, with constant superconducting quantity, taking into account the fact that, at every point of the superconductor, the field should remain less than the critical field, itself connected to the value of the current [54]. Calculation allows specification of the shape and dimensions of this circuit. Even within this "shape" and these "dimensions," it is possible to install, as has been seen (P. 6.2), a winding whose coefficient of self conduction can have any value whatsoever in a very wide range.

Ultimately, the shape and dimensions finally arrived at are generally less "coherent" than those required to produce intense magnetic fields. Likewise, the fields where it has been found appropriate to use a given material are generally less than those used for production of intense magnetic fields and the currents are, on the contrary, higher [54]. /51

Other conditions can still become a factor in the "optimization" of the windings such as, for example, conditions of mechanical resistance ..., or again the fact that it is sought to produce a winding which should not radiate electromagnetically externally to itself, etc.

In this last case, for example, when toric geometries [54] are used, the "optimization" leads to an expression of energy density stored per unit of mass of superconducting material of the form:

$$\frac{w}{m} = k \delta_e \sqrt[3]{B_c} \sqrt[3]{w} \qquad (23)$$

in which: w is the total stored energy,

m the mass of superconductor used,

k a coefficient which is a function of the shape of the toroidal core,

δ_c and B_c respectively the critical current density and critical induction of the material used, both these values corresponding to the same operating point.

There may be deduced from this that the maximum energy density stored increases as the cubic root of the total stored energy W.

By solving for W the maximum total stored energy is obtained:

$$W = k^{3/2} \; m^{3/2} \; \delta_c^{3/2} \; \sqrt[2]{B_c} \qquad (24)$$

and it can be seen that it is not proportional to B_c^2 (as an overly hasty examination of formula (20) could have allowed assumption on a preliminary basis).

In this case, for a given mass of superconductor m, the stored energy W is therefore maximum when the product $\delta_c^{3/2} \sqrt[2]{B_c}$, or what amounts to the same $\delta_c \sqrt[3]{B_c}$, is maximum. /52

(For examination of another aspect of this question, reference can also be made to P. 7.5.1.3.4.)

6.7. How much energy can be stored?

It has been seen that:

- at the present time, the energy stored by means of superconductors could reach in certain cases 10^{10} times the energy that could be stored in the same conventional circuit (P. 6.4);

- with 100 kG there may be produced in the dielectric an energy density of 40 kJ/liter, this energy density increasing as the square of induction B;

- in the case of an optimized circuit (24), the total stored energy increases as the power 3/2 of the mass of superconductor used, this allowing production of energy densities, related to the total superconductor mass, comparable to those obtained in explosives.

Some limitations will become a factor as made necessary by the volume, weight or cost of the device. Still, volume, weight and cost will have a tendency to decrease as progress is made with the materials and they will be better suited for the problem of storage (P. 10). The design of massive devices will considerably simplify manufacture, obviating the requirement for initially preparing a wire, cable or tape, then winding it, etc. Likewise, materials which have up until now not been used owing to difficulties created by preparation of wires or tape will henceforth be available for use. It can even be forecast that when superconductors are produced industrially on a larger scale, their cost price will decrease. /53

The problems involving mechanical stresses on circuits will have to be solved chiefly in high fields since magnetic pressure p increases as the square of induction B:

$$p = \frac{B^2}{2\mu_r\mu_0} \tag{25}$$

When it is possible to solve these problems and use, for example, superconducting materials operating at very high critical fields such as those pointed out in technical literature [85] under the term of materials "with negative fields," huge energy densities will then be produced in the dielectric. For example, at 10^6 G 4 MJ/liter is produced in the dielectric, i.e. an energy density comparable to that contained in explosives. The stored energy density related to the superconducting mass used could probably be still much higher.

7. The Discharge of Energy /56

Since the use circuit is given, the stored energy must be transferred there. Several methods can be used to accomplish this:

7.1. Direct connection:

As was shown by Figure 3, the impedance of use Z_u can be directly connected to terminals A and B of storage coil L (Figure 8).

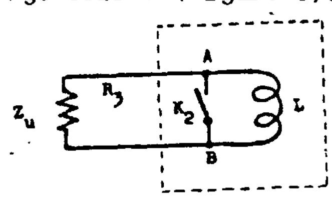


Figure 8.

The problem arising from connections between the low-temperature circuit (enclosed by dotted line in Figure 8) and impedance of use Z_u at ambient temperature is not the same during discharge as the one arising during charging. Indeed, for example, in all cases of high-speed discharges, i.e. greatly damped, and more particularly in cases where the use circuit is essentially dissipative (P. 7.3.1), current I (t) during discharge remains less than initially trapped current I_o, then quickly tends to zero. The Joule effect in connections of resistance R_3 may be expressed.

$$W_d = \int_0^\infty R_3 \; I^2 (t) \; dt \qquad (26)$$ /57

It is then less than

$$R_3 \; I_o^2 \; \tau$$

in which τ is the practical duration of discharge, the duration such that the current becomes practically zero.

This Joule effect, therefore, becomes proportionally weak as the discharge weakens. In very swift discharges it can become extremely weak when suitable precautions are taken so that the "skin effect," well known in high frequencies, does not cause too great an increase of R_3.

In certain cases, impedance of use Z_u can also be located at low temperature which can allow decrease in size of connections or even practically removing these connections from special configuration [74].

In all cases, coil L should be designed in such a manner as to produce the adaptation of given impedance Z_u. It could be possible in some cases to contemplate supplementary switches allowing modification of the impedance shown by storage circuit L during discharge (for example, by using turns which, once charged in series, would be discharged in parallel [86]).

The coil L can be produced by means of "stabilized" material by addition of standard conductors which can then be used to assist the flow of energy towards use at the time of the transitory discharge mode [87] (cf. also P. 7.5.1.3.2 and 7.5.1.3.3).

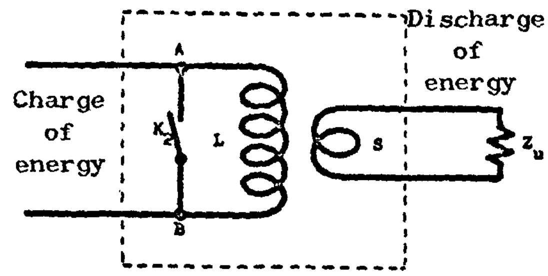
7.2. Inductive coupling: /58

The impedance of use Z_u can also be coupled by induction to the energy storage circuit, either directly: it then itself forms a sort of secondary circuit, either through the intermediary of a secondary circuit S to which it is connected and which is coupled by induction to the storage circuit (Figure 9). The system of coupled circuits then behaves in the same way as a transformer.

In the case of secondary circuit S, it is possible to use a superconducting material and "stabilize" it so as to avoid risks of transitions during transitional modes [13] of discharge. It is also possible to use a standard conducting material with low resistivity (and low magnetoresistance) arranged so as to reduce the skin effect in order for the Joule loss during duration of discharge to be sufficiently low (P. 7.4).

Such a circuit allows either charging or storing energy with currents very different from those with which it is then used. It is also possible to simultaneously couple to the storage circuit several secondary circuits in cases where use includes different circuits.

This circuit "as transformer" allows producing a good adjustment of impedances between the storage circuit and the use circuit. In the case where use Z_u is dissipative, it is thus possible, by taking the necessary precautions (P. 7.3.1) to achieve high efficiencies [55] [88] very close to unity. (Insofar as concerns reference [88], it should nevertheless be pointed out that the writer forgets to show that by modifying the transformer ratio, when all other quantities remain the same, the discharge "time constant" is modified. Cf. P. 7.5.1.3.3)



/59

Figure 9.

7.3. <u>Discharge characteristics</u>: /60

The law of discharge of energy is a function of the nature of impedance of use Z_u and the rule governing opening of switch K_2. We are going to briefly summarize the different effect.

7.3.1. <u>Dissipative use impedance</u>:

When impedance Z_u is essentially dissipative, a damped discharge is produced. In order to carry away the oscillation from the discharge it is necessary for the circuit to have great stray capacities (P. 7.3.4).

Let us, therefore, consider the case in which Z_u is purely dissipative (total conversion of energy into work, into heat, etc.). It behaves in this case like a pure resistance R_u which can be a function of time. By designating by R_2 (t) the resistance shown by switch K_2 as a function of time (such that R_2 (o) = 0) and by granting:

$$R(t) = \frac{R_u(t) \; R_2(t)}{R_u(t) + R_2(t)} \qquad (27)$$

expressions may easily be formed which give: the current i (t) in storage circuit L

$$i(t) = I_o \; e^{-\frac{1}{L}\int_0^t R(t)\,dt} \qquad (28)$$

the voltage at the terminals of resistance of use R_u

$$v(t) = R(t) \; I_o \; e^{-\frac{1}{L}\int_0^t R(t)\,dt} \qquad (29)$$

the derivative of this voltage with respect to time

$$\frac{dv(t)}{dt} = \left[\frac{dR(t)}{dt} - \frac{R^2(t)}{L}\right] I_o \; e^{-\frac{1}{L}\int_0^t R(t)\,dt} \qquad (30)$$ /61

the current in the resistance of use R_u

$$i_u(t) = \frac{R_2(t)}{R_u(t) + R_2(t)} I_o \; e^{-\frac{1}{L}\int_0^t R(t)\,dt} \qquad (31)$$

The relations (29) and (30) show that, when R (t) is a decreasing function, voltage v (t) at the terminals of use starts from zero, passes through a maximum which takes place for one of the values of t voiding the bracket of equation (30), then redescending asymptotically to zero.

The relation (28) shows that when resistance R (t) undergoes swift fluctuations, current i (t) delivered by the storage coil will not have a tendency to follow these fluctuations. On the other hand, the voltage at the terminals of use R_u will then have a tendency according to (29) to vary in the same way as R (t). The storage circuit will then behave as a "current generator" and will be hardly sensitive to fluctuations of the impedance of use.

In the special case in which the resistance of use is constant, we are going to show that instants t_i at which take place the extremes of voltage v (t) at its terminals are independent of its characteristic value R_u. The same is true of current $i_u(t)$ traveling through it.

Instants t_i are values of t among those voiding the bracket of relation (30):

$$\frac{d\,R\,(t)}{dt} - \frac{R^2\,(t)}{L} = 0 \tag{32}$$

equation in which R (t) is expressed according to (27): /62

$$R\,(t) = \frac{R_u\,R_2(t)}{R_u + R_2(t)} \tag{33}$$

By removing R (t), equation (32) becomes, when R_u is not zero:

$$\frac{d\,R_2\,(t)}{dt} - \frac{R_2^2\,(t)}{L} = 0 \tag{34}$$

Since this equation does not involve the resistance of use R_u, its roots are quite independent of R_u.

In the special case in which equation (34) has only one root, which we shall designate by t_1, (i.e. when R_2 (t), for example, is a monotonous non-decreasing function such that its first derivative is itself a non-decreasing monotonous function) and in the case in which R_2 (t) reaches, from the beginning of discharge, values much greater than R_u, the current i_u (t) given by relation (31) follows a law whose general aspect can be diagrammed by Figure 10.

At beginning of discharge, so long as R_2 (t) remains much less than R_u, relation (33) is equivalent to:

$$R\,(t) \sim R_2\,(t) \tag{35}$$

The slope at the origin of the rise front of the voltage may be expressed according to (30):

$$\left(\frac{d\,v(t)}{dt}\right)_{t=0} = I_o\,\frac{d\,R_2\,(t)}{dt} \tag{36}$$

whence there follows directly for the current in the use: /63

$$\left(\frac{d\,i_u(t)}{dt}\right)_{t=0} = \frac{I_o}{R_u}\;\frac{d\,R_2(t)}{dt} \tag{37}$$

At beginning of discharge relations (29) and (31) become, taking (35) into account:

$$v\,(t) \sim R_2\,(t)\,I_o\;e^{-\frac{1}{L}\int_0^t R_2\,(t)\,dt} \tag{38}$$

$$i_u\,(t) \sim R_2\,(t)\,\frac{I_o}{R_u}\;e^{-\frac{1}{L}\int_0^t R_2\,(t)\,dt} \tag{39}$$

The result is that when function R_2 (t) undergoes swift fluctuations on the scale of the period of time considered, these fluctuations will then be faithfully reproduced by the functions representing voltage v (t) and current $i_u(t)$.

Consequently, the beginning of the pulse rise front can be considered as very representative of the switch characteristics.

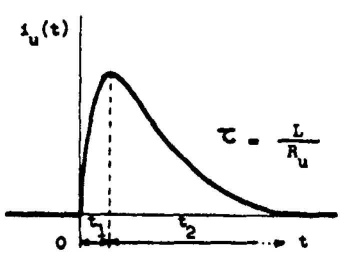


Figure 10.

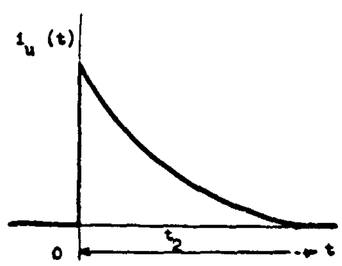


Figure 11.

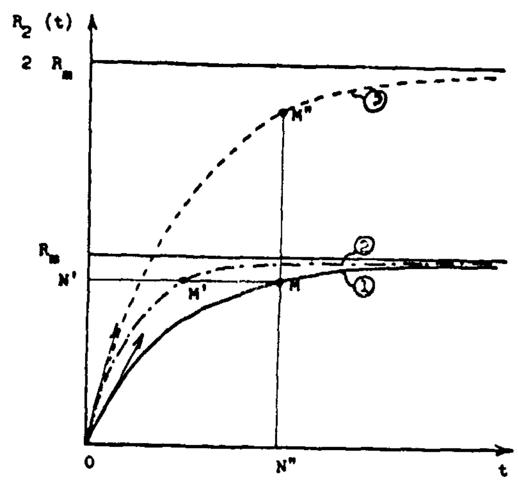
The duration t_2, in the case where R_2 (t) reaches, from the very beginning of discharge, values much greater than R_u, is only a function of time constant $\tau = \frac{L}{R_u}$ of the storage circuit closed upon use. /64

When the switch gains a resistance much greater than R_u in a very short time before $\frac{L}{R_u}$, there should be produced, for all practical purposes, a law of discharge such as the one diagrammed by Figure 11, representing equation:

$$i_u\,(t) = I_o\,e^{-\frac{R_u}{L}t} \tag{40}$$

which is consequent on (31).

When the impedance of use is given, provided the switch is suitable (P. 7.5), the rate of discharge is essentially a function of the storage circuit coil. Now, it has been seen (P. 6.2) that it was possible to store a given energy with any coil at all with specific limits. It will therefore always be possible to produce the storage circuit in such a manner as to produce a given "discharge rate" within corresponding limits. The possibilities will be still more widespread when an "as transformer" circuit (P. 7.2) is used for the discharge.



/65

Figure 12.

Display of function $R_2\ (t) = R_m\ (1 - e^{-kt})$

- curve (1) shows function $R_2\ (t) = R_m\ (1 - e^{-kt})$ of which the derivative is $\frac{d}{dt}\ R_2(t) = R_m\ ke^{-kt}$;

- curve (2) shows function $R_2\ (t) = R_m\ (1 - e^{-\alpha kt})$ (It is deduced from (1) by a contraction ($\frac{M'N'}{M\ N'} = \frac{1}{\alpha}$) parallel to the time axis.);

- curve (3) shows function $R_2\ (t) = \alpha\ R_m\ (1 - e^{kt})$ (It is deduced from (1) by an expansion ($\frac{M''N''}{MN''} = \alpha$) parallel to the axis of resistances.);

- the functions (2) and (3) have the same derivative: $\frac{d}{dt}\ R_2\ (t) = \alpha\ R_m ke^{-kt}$; on the figure, α has been taken equal to 2.

7.3.1.1. Examination of a special case category: /66

We are going to make a more detailed examination of a specific category of cases which involves many applications (P. 8), in which:

- use is equivalent to a constant R_u resistance;

- impedance of the connections may be disregarded;

- resistance as a function of time of the switch, during its opening, obeys a law in the form:

$$R_2\ (t) = R_m\ (1 - e^{-kt}) \qquad (41)$$

e being the base of Naperian logarithms,

R_m being consequently the final resistance of the switch in the normal state,

and k a constant quantity characterizing the rate of transition.

Figure .2 shows the effect of k and R_m on the behavior of this function whose more precise definition will be found in P. 7.5.1.2.

A law in this form has already been used [93] and appears to agree with those of our experiments with which it was compared [95].

7.3.1.1.1. Rise front of the pulse:

We are going to calculate the value of t_1 which corresponds to the maximum of current (Figure 10) in the resistance of use, i.e. with maximum voltage at its terminals.

The bracket of relation (30) becomes, by replacing in (27) the expression /67 R_u (t) by the constant resistance of use R_u and expression R_2 (t) by the relation (41):

$$\left[\frac{d\,R\,(t)}{dt} - \frac{R^2\,(t)}{L}\right] = \frac{R_u^2\;R_m^2}{L\,(R_u + R_m - R_m e^{-kt})^2}\left[-e^{-2\,kt} + \left(2 + \frac{kL}{R_m}\right) e^{-kt} - 1\right] \tag{42}$$

The first factor is positive on the right hand side of the equation. The factor between brackets is a polynomial of the second degree with e^{-kt} which is positive for t = 0, decreases when t increases, is reduced to zero for value t_1 given in (44) and is negative for t less than t_1. Consequently, the function v (t) given by (29) passes through a maximum for $t = t_1$.

If it is true that:

$$\frac{L}{R_m} = \tau_2 \tag{43}$$

(τ_2 is the standard time constant characteristic of the storage circuit, i.e. the time constant of discharge of the storage coil on the switch alone entirely in the normal state (constant R_m resistance and impedance of use being disconnected).

It follows that:

$$t_1 = -\frac{\lambda}{k}\,\mathrm{Log}\left[1 + \frac{k\tau_2}{2} - \sqrt{\left(1 + \frac{k\tau_2}{4}\right) k\tau_2}\right] \tag{44}$$

The curves of Figure 12-1 show the variation of the term between brackets, this term being designated by y_1 (consistant with relation (55)).

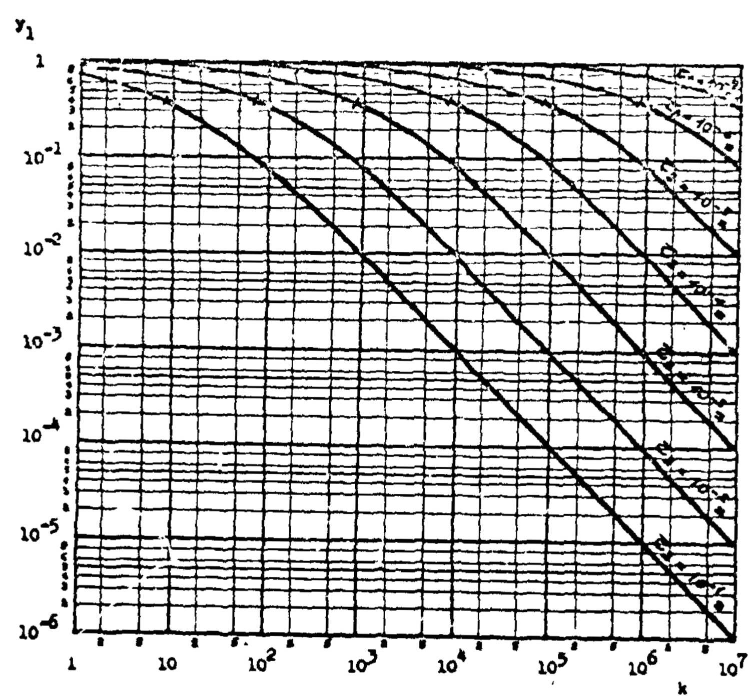
The curves of Figure 12.2 show the variation of t_1.

Variation of /68

$$y_1 = \left[1 + \frac{k\tau_2}{2} - \sqrt{\left(1 + \frac{k\tau_2}{4}\right) k\tau_2}\,\right]$$

as a function of k for different values of τ_2

(Heavy line: part of the curves corresponding to $t_1 < \tau_2$)



second^{-1}

Figure 12.1.

Variation of /69

$$t_1 = -\frac{1}{k} \text{Log } y_1$$

as a function of k for different values of τ_2

(Heavy line: part of the curves corresponding to $t_1 < \tau_2$)

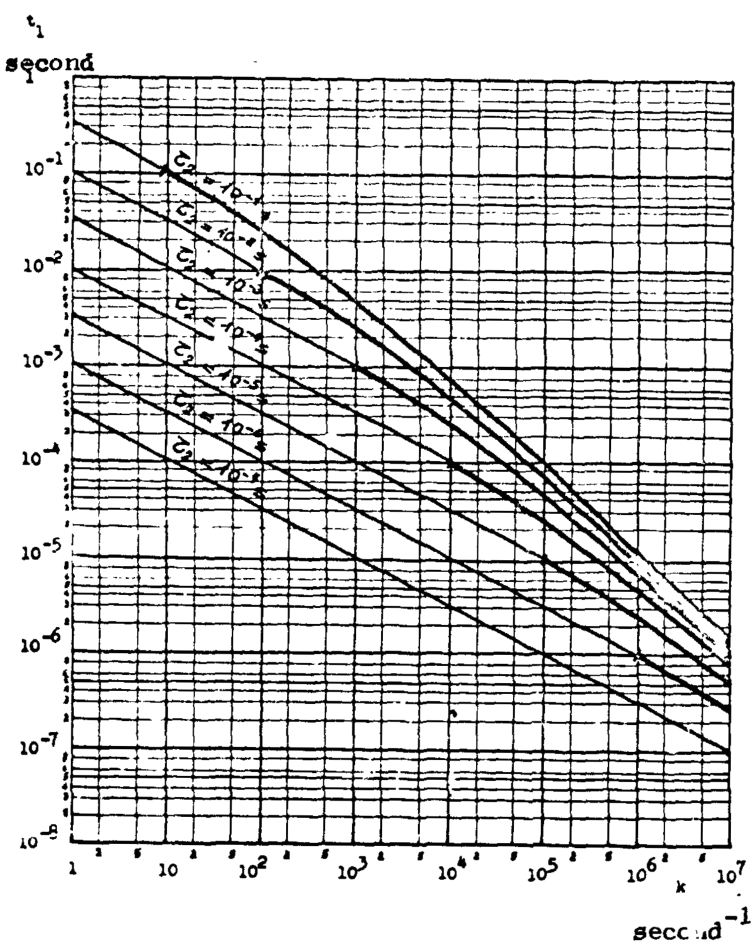


Figure 12.2.

These curves were plotted as a function of k for different values of τ_2. /70
The part of the curves corresponding to $t_1 < \tau_2$ has been plotted more boldly.

When $k\tau_2$ has an order of magnitude much less than unity, this expression becomes:

$$t_1 \sim \sqrt{\frac{\tau_2}{k}} \tag{45}$$

Relations (41) and (44) provide for $t = t_1$, the corresponding value of the switch resistance:

$$R_2(t_1) = R_m \left[\sqrt{\left(1 + \frac{k\tau_2}{4}\right) k\tau_2} - \frac{k\tau_2}{2}\right] \tag{46}$$

When $k\tau_2 \ll 1$, it follows that:

$$R_2(t_1) \sim R_m \sqrt{k\tau_2} \tag{47}$$

Relations (29) and (31) provide respectively for $t = t_1$ the values of the maximums of voltage and current in **the resistance of use.**

The relations (36) and (37) provide respectively for $t = 0$, the slope at the origin of the rise fronts of the voltage and current in use:

$$\left(\frac{d\,v(t)}{dt}\right)_{t=0} = k\,R_m\,I_0 \tag{48}$$

$$\left(\frac{d\,i_u(t)}{dt}\right)_{t=0} = k\,\frac{R_m}{R_u}\,I_0 \tag{49}$$

It can be advantageous to compare:

- the absolute value of the slope at the origin of the rise front of the current pulse,

- and the slope which its descending part (given by (40) and shown by dotted line on Figure 13) would have had at the origin if the switch had reached an infinite resistance in a time practically zero (perfect switch P. 7.5).

If it is true that:

$$\frac{L}{R_u} = \tau \tag{50}$$

the slope at the origin of function (40) is:

$$\left(\frac{d\,i_u(t)}{dt}\right)_{t=0} = -\frac{I_0}{\tau} \tag{51}$$

The comparison of the absolute values of both these slopes amounts to comparing the absolute value of expressions (49) and (51), i.e. by taking (43) and (50) into account:

$$k\,\frac{\tau}{\tau_2} \quad \text{and} \quad \frac{1}{\tau}\,. \tag{52}$$

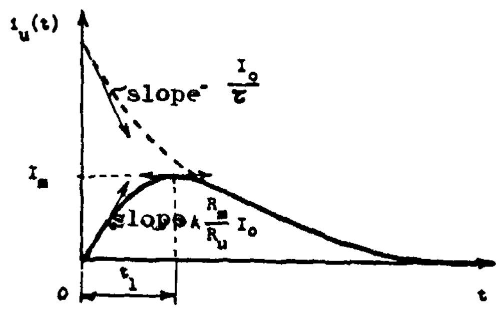


Figure 13

7.3.1.1.2. Energy transferred in use resistance: /72

The energy transferred during discharge up until instant t, in the resistance of use, may be expressed:

$$W(t) = \int_0^t \frac{v^2(t)}{R_u} dt \tag{53}$$

and by replacing v(t) by its expression (29):

$$W_u(t) = \frac{I_o^2}{R_u} \int_0^t R^2(t)\, e^{-\frac{2}{L}\int_0^t R(t)\,dt} dt \tag{54}$$

Taking (33) and (41) into account and if it is true that:

$$e^{-kt} = y \tag{55}$$

and

$$\frac{R_m + R_u}{R_m} = a \tag{56}$$

it follows that:

$$W_u(t) = -\frac{R_u I_o^2}{a} \int_1^{y(t)} \frac{(1-y)^2}{y(a-y)^2} e^{\frac{2R_u}{kL}\int_1^y \frac{1-y}{y(a-y)} dy} dy \tag{57}$$

From the relation:

$$\int_1^y \frac{1-y}{y(a-y)} dy = \int_1^y \frac{dy}{y(a-y)} - \int_1^y \frac{dy}{a-y} = -\frac{1}{a} \operatorname{Log} \frac{a-y}{y(a-1)} + \operatorname{Log} \frac{a-y}{a-1} \tag{57.1}$$

it follows that:

$$e^{b\int_1^y \frac{1-y}{y(a-y)} dy} = \left(\frac{a-y}{y(a-1)}\right)^{-\frac{b}{a}} \left(\frac{a-y}{a-1}\right)^b \tag{57.2}$$

and expression (57) becomes: [97] /73

$$W_u(t) = W_o\, b(a-1)^{-b\frac{a-1}{a}} \int_{y(t)}^1 (1-y)^2 (a-y)^{b\frac{a-1}{a}-2} y^{\frac{b}{a}-1} dy \tag{58}$$

if it is true that:

$$\frac{1}{2} L I_o^2 = W_o \tag{58.1}$$

$$\frac{R_u}{k L} = b \tag{59}$$

We are going to carry out the calculation of relation (58) by using a development in series of the express which is found under the integration sign. [97]

We are going to assume:

$$b\,\frac{a-1}{a} - 2 = \alpha \qquad (59.1)$$

$$\frac{b}{a} - 1 = \beta \qquad (59.2)$$

We can then write:

$$(1-y)^{\alpha} = \begin{cases} a^{\alpha} - \alpha\, a^{\alpha-1}\, y + \frac{\alpha(\alpha-1)}{2!}\, a^{\alpha-2}\, y^{2} - \frac{\alpha(\alpha-1)(\alpha-2)}{3!}\, a^{\alpha-3}\, y^{3} \\ + \ldots\ldots + (-1)^{p}\, \frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}\, a^{\alpha-p}\, y^{p} + \ldots \end{cases} \qquad (59.3)$$

and since:

$$(1-y)^{2} = 1 - 2y + y^{2} \qquad (59.4)$$

The general term of the development in series of the expression under the integration sign, in relation (58) is written:

$$y^{p+\beta}\left[(-1)^{p}\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}\, a^{\alpha-p} + 2(-1)^{p}\frac{\alpha(\alpha-1)\ldots(\alpha-p+2)}{(p-1)!}\, a^{\alpha-p+1} + (-1)^{p}\frac{\alpha(\alpha-1)\ldots(\alpha-p+3)}{(p-2)!}\, a^{\alpha-p+2}\right] \qquad (59.5)$$

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By paying attention to the significance of the operations which it expresses, this relation is valid beginning from p = 3. The three first terms of the development in series, corresponding to the powers of y with β, $\beta + 1$ and $\beta + 2$, are easily obtained beginning from relations (59.3) and (59.4).

A primitive element of the general term (59.5) is:

$$(-1)^{p}\,\frac{y^{p+\beta+1}}{p+\beta+1}\,\frac{\alpha(\alpha-1)\ldots(\alpha-p+3)}{(p-2)!}\, a^{\alpha-p}\left[\frac{(\alpha-p+2)(\alpha-p+1)}{(p-1)p} + 2\,\frac{\alpha-p+2}{p-1}\, a + a^{2}\right] \qquad (59.6)$$

Integration (58) leads to the taking of the value of this primitive element for y = 1 reduced by the value for y = y(t).

The development of the integral appearing in relation (58) is then written:

1st term: $(1-y^{\beta+1})\,\frac{a^{\alpha}}{\beta+1}$

2nd term: $-(1-y^{\beta+2})\,\frac{1}{\beta+2}\,(\alpha\, a^{\alpha-1} + 2\, a^{\alpha})$

3rd term: $(1-y^{\beta+3})\,\frac{1}{\beta+3}\,\left(\frac{\alpha(\alpha-1)}{2!}\, a^{\alpha-2} + 2\alpha\, a^{\alpha-1} + a^{\alpha}\right)$

4th term: $-(\cdot-y^{\beta+4})\,\frac{1}{\beta+4}\,\left(\frac{\alpha(\alpha-1)(\alpha-2)}{3!}\, a^{\alpha-3} + 2\,\frac{\alpha(\alpha-1)}{2!}\, a^{\alpha-2} + \alpha\, a^{\alpha-1}\right)$

p^{-nth} teim: $(-1)^{p-1}\,(1-y^{\beta+p})\,\frac{1}{\beta+p}\; A$

with $A = \left(\frac{\alpha(\alpha-1)\ldots(\alpha-p+2)}{(p-1)!}\, a^{\alpha-p+1} + 2\,\frac{\alpha(\alpha-1)\ldots(\alpha-p+3)}{(p-2)!}\, a^{\alpha-p+2} + \frac{\alpha(\alpha-1)\ldots(\alpha-p+4)}{(p-3)!}\, a^{\alpha-p+3}\right)$

By arranging the sum of these terms according to the powers of a, the following series may be obtained: . /75

$$\gamma(y(t)) = a^{\alpha}\left(\frac{1-y^{\beta+1}}{\beta+1} - \frac{2(1-y^{\beta+2})}{\beta+2} + \frac{1-y^{\beta+3}}{\beta+3}\right) \tag{59.7}$$

$$- a^{\alpha-1}\alpha\left(\frac{1-y^{\beta+2}}{\beta+2} - \frac{2(1-y^{\beta+3})}{\beta+3} + \frac{1-y^{\beta+4}}{\beta+4}\right)$$

$$+ a^{\alpha-2}\frac{\alpha(\alpha-1)}{2!}\left(\frac{1-y^{\beta+3}}{\beta+3} - \frac{2(1-y^{\beta+4})}{\beta+4} + \frac{1-y^{\beta+5}}{\beta+5}\right)$$

$$+ (-1)^{p}\, a^{\alpha-p}\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}\left(\frac{1-y^{\beta+p+1}}{\beta+p+1} - \frac{2(1-y^{\beta+p+2})}{\beta+p+2} + \frac{1-y^{\beta+p+3}}{\beta+p+3}\right)$$

By multiplying the sum of this series by the factor $W_o\, b(a-1)^{-b\frac{a-1}{a}}$, there is produced the value of expression (58) of $W_u(t)$ at instant t:

$$W_u(t) = W_o\, b(a-1)^{-b\frac{a-1}{a}}\,\gamma(y(t)) \tag{59.8}$$

In the case of $t = \infty$, the result from (55) is that the corresponding value of y is zero and series (59.7) is then written: /76

$$\gamma = \frac{2}{(\beta+1)(\beta+2)(\beta+3)}\, a^{\alpha} \tag{59.9}$$

$$- \frac{2}{(\beta+2)(\beta+3)(\beta+4)}\, a^{\alpha-1}\,\alpha$$

$$+ \frac{2}{(\beta+3)(\beta+4)(\beta+5)}\, a^{\alpha-2}\,\frac{\alpha(\alpha-1)}{2!}$$

$$+ (-1)\, \frac{2}{(\beta+p+1)(\beta+p+2)(\beta+p+3)}\, a^{\alpha-p}\,\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}$$

By multiplying the sum of this series by the factor $W_o\, b(a-1)^{-b\frac{a-1}{a}}$, the value of expression (58) of $W_u(t)$ is produced for $t = \infty$, i.e., in reality, the energy which has been transferred in the resistance of use during discharge:

$$W_u(\infty) = W_o\, b\,(a-1)^{-b\frac{a-1}{a}}\,\gamma \tag{59.10}$$

7.3.1.1.3. Energy dissipated in the switch:

The energy dissipated during discharge up until instant t in the switch may be expressed:

$$w_2(t) = \int_0^t \frac{v^2(t)}{R_2(t)}\,dt \tag{60}$$

and by replacing v(t) by its expression (29):

$$w_2(t) = I_o^2 \int_0^t \frac{R^2(t)}{R_2(t)}\, e^{-\frac{2}{L}\int_0^t R(t)\,dt}\,dt \tag{61}$$

By taking into account (33), (41), (55), (56), (58.1) and (59), it follows that by carrying out a first integration in the same manner as in paragraph 7.3.1.1.2 [97]: /77

$$w_2(t) = W_o\,\frac{R_u}{R_m}\, b\,(a-1)^{-b\frac{a-1}{a}} \int_{y(t)}^{1} (1-y)(a-y)^{b\frac{a-1}{a}-2}\, y^{\frac{b}{a}-1}\,dy \tag{62}$$

We are likewise going to carry out the calculation of this relation by using the development in series of the expression found under the integration sign. Using the same notations as in paragraph 7.3.1.1.2, the general term of this development in series is written:

$$(-1)^{p}\, y^{\beta+p}\, \frac{\alpha(\alpha+1)\ldots(\alpha-p+2)}{(p-1)!}\, a^{\alpha-p}\left[\frac{\alpha-p+1}{p}+a\right] \tag{62.1}$$

A primitive element of this expression is:

$$(-1)^{p}\, \frac{y^{\beta+p+1}}{\beta+p+1}\, \frac{\alpha(\alpha-1)\ldots(\alpha-p+2)}{(p-1)!}\, a^{\alpha-p}\left[\frac{\alpha-p+1}{p}+a\right] \tag{62.2}$$

The integration (62) suggests taking the value of this primitive for $y = 1$ reduced by the value for $y = y(t)$.

The development of the integral appearing in relation (62) is then written:

1st term: $(1-y^{\beta+1})\,\frac{a^{\alpha}}{\beta+1}$

2nd term: $-(1-y^{\beta+2})\,\frac{1}{\beta+2}\,(\alpha a^{\alpha-1}+a^{\alpha})$

3rd term: $(1-y^{\beta+3})\,\frac{1}{\beta+3}\,\left(\frac{\alpha(\alpha-1)}{2!}a^{\alpha-2}+\alpha a^{\alpha-1}\right)$

4th term: $-(1-y^{\beta+4})\,\frac{1}{\beta+4}\,\left(\frac{\alpha(\alpha-1)(\alpha-2)}{3!}a^{\alpha-3}+\frac{\alpha(\alpha-1)}{2!}a^{\alpha-2}\right)$

p^{nth} term: $(-1)^{p-1}(1-y^{\beta+p})\,\frac{1}{\beta+p}\,\left(\frac{\alpha(\alpha-1)\ldots(\alpha-p+2)}{(p-1)!}a^{\alpha-p+1}+\frac{\alpha(\alpha-1)\ldots(\alpha-p+3)}{(p-2)!}a^{\alpha-p+2}\right)$ /78

By arranging the sum of these terms according to the powers of a the following series is produced:

$$\begin{aligned}
S(y(t)) = {} & a^{\alpha}\left(\frac{1-y^{\beta+1}}{\beta+1}-\frac{1-y^{\beta+2}}{\beta+2}\right)\\
& - a^{\alpha-1}\alpha\left(\frac{1-y^{\beta+2}}{\beta+2}-\frac{1-y^{\beta+3}}{\beta+3}\right)\\
& + a^{\alpha-2}\frac{\alpha(\alpha-1)}{2!}\left(\frac{1-y^{\beta+3}}{\beta+3}-\frac{1-y^{\beta+4}}{\beta+4}\right)\\
& \ldots\\
& + (-1)^{p} a^{\alpha-p}\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}\left(\frac{1-y^{\beta+p+1}}{\beta+p+1}-\frac{1-y^{\beta+p+2}}{\beta+p+2}\right)
\end{aligned} \tag{62.3}$$

By multiplying the sum of this series by the factor $W_o \frac{R_u}{R_m} b\,(a-1)^{-b\frac{a-1}{a}}$, the value of expression (62) of $W_2(t)$ at instant t is produced:

$$W_2(t) = W_o \frac{R_u}{R_m}\, b\,(a-1)^{-b\frac{a-1}{a}}\, S(y(t)) \tag{62.4}$$

In the case of $t = \infty$, the result from (55) is that the corresponding value of y is zero and the series (62.3) is then written:

$$\begin{aligned}
S = {} & \frac{1}{(\beta+1)(\beta+2)}\, a^{\alpha}\\
& - \frac{1}{(\beta+2)(\beta+3)}\, a^{\alpha-1}\alpha\\
& + \frac{1}{(\beta+3)(\beta+4)}\, a^{\alpha-2}\,\frac{\alpha(\alpha-1)}{2!}\\
& \ldots\ldots\ldots\ldots\\
& + (-1)^{p}\,\frac{1}{(\beta+p+1)(\beta+p+2)}\, a^{\alpha-p}\,\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!}
\end{aligned} \tag{62.5}$$

By multiplying the sum of this series by the factor $W_o \frac{R_u}{R_m} b\,(a-1)^{-b\frac{a-1}{a}}$, /79
the value of expression (62) of $W_2(t)$ is produced for $t = \infty$, i.e., in reality, the energy which has been dissipated in the switch during discharge:

$$W_2(\infty) = W_o \frac{R_u}{R_m} b\,(a-1)^{-b\frac{a-1}{a}} S \tag{62.6}$$

7.3.1.1.4. <u>Expression of the current in use resistance:</u>

Relation (31) is written, still using notations provided in (55), (56) and (59):

$$i_u(t) = \frac{1-y}{a-y}\, I_o\, e^{\frac{b}{2}\int_1^{y(t)} \frac{1-y}{y(a-y)}\,dy} \tag{62.7}$$

then, by using relation (57.2), it follows that:

$$i_u(t) = I_o\,(a-1)^{\frac{b}{2}\frac{1-a}{a}}\,(1-y)\,(a-y)^{\frac{b}{2}\frac{a-1}{a}-1}\,y^{-\frac{b}{2a}} \tag{62.8}$$

The maximum value I_m of i_u (Figure 13) occurs at instant $t = t_1$ provided by relation (44). It is then true that:

$$I_m = i_u(t_1) \tag{62.9}$$

7.3.2. <u>Coil use impedance:</u> /80

We are going to see that when impedance Z_u is essentially of a coil nature, the energy transfer is, on the contrary, unfavorable.

Let us examine the extreme case in which Z_u behaves as a pure coil L_u (for example, winding intended exclusively to create a magnetic field and producing neither work nor heat...) (Figure 14), switch K_2 being not of a coil nature.

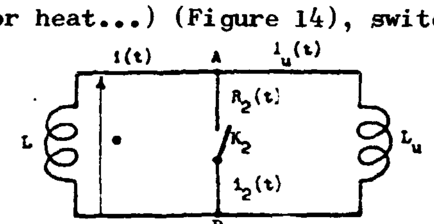


Figure 14.

The Kirchoff equations:

$$\begin{cases} i(t) = i_u(t) + i_2(t) & (63) \\ 0 = -L\frac{di(t)}{dt} = R_2(t)\,i_2(t) = L_u\frac{di_u(t)}{dt} & (64) \end{cases}$$

provide:

the current in the switch:

$$i_2(t) = I_o\, e^{-\frac{L+L_u}{L L_u}\int_0^t R_2(t)\,dt} \tag{65}$$

the current in the use: /81

$$i_u(t) = \frac{L}{L+L_u}\, I_o \left(1 - e^{-\frac{L+L_u}{L L_u}\int_0^t R_2(t)\,dt}\right) \tag{66}$$

By designating W_o (with $W_o = 1/2\, LI_o^2$) the initial energy stored in L, these expressions allow calculation of:

the magnetic energy transferred into L_u:

$$W_u = \frac{1}{2} L_u i_u^2 (\infty) = W_o \frac{L L_u}{(L + L_u)^2} \quad (67)$$

(No matter what the value of L may be, this expression cannot be greater than $\frac{W_o}{4}$.)

the magnetic energy remaining in L after discharge:

$$W = \frac{1}{2} L i^2(\infty) = W_o \frac{L^2}{(L + L_u)^2} \quad (68)$$

the energy dissipated by Joule effect in the switch:

$$W_2 = \int_o^\infty R_2 (t)\; i_2^2\; (t)\; dt = W_o \frac{L_u}{L + L_u} \quad (69)$$

It is rather noteworthy that these quantities of energy, and more particularly the energy dissipated by the Joule effect in the switch, are not a function of the resistance of the switch. They are only a function of the initial energy and values of the coils. The resistance of the switch only plays a role on the rate of discharge (equations (65) and (66)).

In the special case in which the switch can reach, within a negligible period of time, a resistance R_m which remains constant, the current in the switch may be expressed, in accordance with (65): /82

$$i_2 (t) = I_o\, e^{-\frac{R_m}{\lambda} t} \quad (70)$$

by stating $\lambda = \frac{LL_u}{L+L_u}$. The time constant of discharge $\frac{\lambda}{R_m}$ is then the same as what would be obtained if coils L and L_u were discharged in parallel into resistance R_m.

In all cases, since the value of W_2 given by equation (69) is never zero when W_o and L_u are never zero, it follows that in any transfer of coil to coil energy, there is always degradation of a specific quantity of energy in the form of heat no matter what switch is used.

7.3.3. Capacitive use impedance:

We are going to examine the extreme case in which the impedance of use behaves as a pure capacity C_u (Figure 15), since switch K_2 is not of a coil nature and acquires a constant resistance R_m in a negligible time.

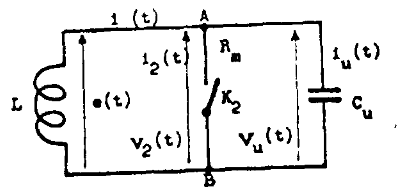


Figure 15.

By respectively designating by $v(t)$, $v_2(t)$, $v_u(t)$ the voltages at the terminals of the coil, switch and capacitor, the Kirchoff equations: /83

$$\begin{cases} i(t) = i_2(t) + i_u(t) & (71) \\ e(t) = -v(t) = v_2(t) = v_u(t) & (72) \end{cases}$$

allow setting up the equation for circuit or eration in the form:

$$C_u \frac{d\,v(t)}{dt} + \frac{v(t)}{R_2} + \frac{1}{L}\int v(t)\,dt = 0 \qquad (73)$$

It may be directly deduced from this equation that voltage $v(t)$ at the terminals of the conductor does not oscillate when

$$R_m < \frac{1}{2}\sqrt{\frac{L}{C_u}} \qquad (74)$$

does oscillate when

$$R_m > \frac{1}{2}\sqrt{\frac{L}{C_u}} \qquad (75)$$

and in the case in which R_2 is very high, the period of oscillation tends toward:

$$\tau = 2\pi\sqrt{L\,C_u} \qquad (76)$$

<u>Comment 1</u>:

When, using Figure 15, the resistance R_m of the switch is replaced by the resistance R given by relation (27), then, when the latter is replaced by a constant resistance R_u, we are now dealing with the case where a constant resistive charge R_u has been switched in between A and B, in parallel with a switch whose resistance $R_2(t)$ to opening becomes very great with respect to R_u in a negligible time period. Relation (75) shows then that, for the discharge to oscillate, it will be necessary for: /84

$$\sqrt{\frac{L}{C_u}} < 2\,R_u \qquad (77)$$

This condition will not be achieved when R_u and C_u are sufficiently small. It follows that energy storage in coils is especially well suited for production of damped discharges when impedance of use is equivalent to a low resistance.

Comment 2:

The case which has just been discussed in Comment 1 can be very simply compared to the case, where consideration could be given on a dual basis, involving energy storage in a capacitor C, the use being established by a constant resistance R_u and the circuit containing an extra coil L_u, Figure 16.

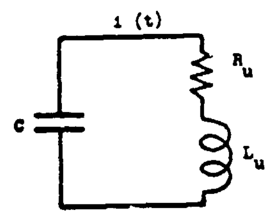


Figure 16.

The operational equation of such a circuit is in the form: /85

$$L_u \frac{d\, i(t)}{dt} + R_u \; i(t) + \frac{1}{C} \int i(t) \; dt = 0 \tag{78}$$

and allows a non-oscillating solution i(t) when:

$$R_u > 2 \sqrt{\frac{L_u}{C}} \tag{79}$$

and an oscillating solution when:

$$R_u < 2 \sqrt{\frac{L_u}{C}} \tag{80}$$

the period of oscillation tending, when R_u is very small, towards the value:

$$\tau = 2\pi \sqrt{L_u \, C} \tag{81}$$

Relation (80) shows that this circuit is poorly suited for producing damped discharges when use is equivalent to a low resistance and this becomes more and more accentuated as a greater capacity C is used. On the other hand, when large capacities are used (and this has been found more appropriate to do when great energies are to be stored, cf. P. 8.1.2, for example) relation (81) shows that it is not possible to obtain low value periods of oscillation. As has been seen in comment 1, storage of energy in coils has no disadvantages.

7.3.4. A given use impedance: /86

Owing to its importance for applications (P. 8), we are going to consider the special case in which the impedance of use has in addition to a dissipative term, a coil term as a function of time. We shall confine ourselves to the ideal conditions of one circuit without stray capacities and with a perfect switch (P. 7.5). Nevertheless, we shall introduce an extra resistance R_3 as well as an extra self induction L_3 into the connections (Figure 17).

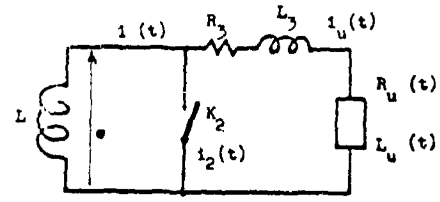


Figure 17.

Using the notations of Figure 15, the perfect switch can be portrayed by the conditions:

in the case of $t < 0$ $\quad \begin{cases} i_2(t) = i(t) \\ i_u(t) = 0 \end{cases}$

in the case of $t \geqslant 0$ $\quad \begin{cases} i_2(t) = 0 \\ i_u(t) = i(t) \end{cases}$

Let us, therefore, consider the discharge of energy into the circuit of use (i.e. $t \geq 0$). The Kirchoff equations are then reduced to the single equation:

$$0 = -L \frac{d\,i(t)}{dt} - \left[R_3 + R_u(t)\right] i(t) + L_3 \frac{d\,i(t)}{dt} + \frac{d\,\Phi(t)}{dt}$$

In this expression, $\Phi(t)$ is the magnetic flux encompassing the use circuit. It may be expressed in the case concerning us:

$$\Phi(t) = L_u(t) \cdot i(t)$$

The expression (84) is then written:

$$\left[L + L_u(t) + L_3\right] \frac{d\,i(t)}{dt} + \left[R_u(t) + R_3 + \frac{d\,L_u(t)}{dt}\right] i(t) = 0$$

Bu multiplying by $i(t)\,dt$, both sides of this expression, which depict the differences in potential, there is caused to appear a relationship between the energies which become a factor in the period of time $(t, t + dt)$:

$$\frac{1}{2}\left[L + L_u(t) + L_3\right] d\,i^2(t) + \left[R_u(t) + R_3 + \frac{d\,L_u(t)}{dt}\right] i^2(t)\,dt = 0$$

The term $1/2\ L\ di^2(t)$ shows, at instant t, the variation of magnetic energy stored by the superconducting circuit, then travelled through by a current with well defined value $i(t)$, when the current varies by di.

The term $1/2\ L_u\ (t)\ di^2(t)$ shows at instant t, the variation of magnetic energy stored by the use circuit whose coil has a well defined value $L_u\ (t)$, then travelled through by a current with well defined value $i(t)$, when the current varies by di.

These three terms show, at instant t, the variations of magnetic energies, respectively in the storage circuit, in the use circuit and in the connections, which have resulted from the variations of the corresponding magnetic fields, exclusively owing to a variation of the current.

The term $R_u(t)\, i^2(t)\, dt$ depicts the energy dissipated in the use circuit, travelled through by the current with well defined value i(t), between instants t and t + dt.

The term $\frac{dL_u(t)}{dt}\, i^2(t)\, dt$ depicts the energy involved in the use circuit, travelled through by the current with well defined value i(t), as a consequence of its variation of coil $dL_u(t)$ between instants t and t + dt. This term corresponds to the sum of a magnetic energy $1/2\, \frac{dL_u(t)}{dt}\, i^2(t)\, dt$ stored in the coil $dL_u(t)$ and a work $1/2\, \frac{dL_u(t)}{dt}\, i^2(t)\, dt$ created by deformation of the use circuit.

The term $R_3\, i^2(t)\, dt$ depicts the energy dissipated into the connections travelled through by the current i(t) between instants t and t + dt.

The general case of an absolutely given use impedance would go beyond the scope of this article and will not be examined. For practical purposes, the cases that will be encountered will often have predominant characteristics that more or less associate them with one of the cases considered. In this way, it will be very quickly possible, as a first approximation, to understand the general aspect of the phenomena. A more detailed analysis will then be necessary.

7.4. Efficiency of discharge:

The efficiency is the ratio between the transferred energy with use and the initially stored energy. We are going to estimate it in several special cases.

7.4.1. Dissipative use impedance (transformation of energy into work, or into heat, etc....)

7.4.1.1. Expression of the efficiency:

When the impedance of the connections is negligible, the efficiency can be written by designating by v(t) the voltage at the terminals of use R_u and by using (27):

$$\eta = \frac{W_u(\infty)}{W_0} = \frac{\int_0^\infty \frac{v^2(t)}{R_u(t)}\,dt}{\frac{1}{2} L I_0^2} = \frac{\int_0^\infty \frac{v^2(t)}{R_u(t)}\,dt}{\int_0^\infty \frac{v^2(t)}{R(t)}\,dt} = \frac{1}{1 + \frac{\int_0^\infty \frac{v^2(t)}{R_2(t)}\,dt}{\int_0^\infty \frac{v^2(t)}{R_u(t)}\,dt}}$$

In the most special case where R_u is constant, (P. 7.3.1), if R_m designates the upper boundary of $R_2(t)$ during discharge, there may be obtained successively:

$$\eta = \frac{1}{1 + \dfrac{\int_0^\infty \frac{v^2(t)}{R_2(t)}\,dt}{\int_0^\infty \frac{v^2(t)}{R_u}\,dt}} \leqslant \frac{1}{1 + \dfrac{R_u}{R_m}} = \frac{1}{1 + \dfrac{\tau_2}{\tau}}$$

the last equality resulting from (50) and (43) in which R_m designated generally the upper boundary of $R_2(t)$.

The result is that:

- the efficiency will always be less at the upper boundary represented by the right hand side of the inequality (89);

- this upper boundary will increase in height as R_m increases with respect to R_u, i.e. the standard characteristic time constant τ_2 of the storage circuit will be lower with respect to the discharge time constant τ;

- this upper boundary of the efficiency will be better reached when $R_2(t)$ more swiftly reaches its maximum value. Indeed, the expression completely to the right of (89) is the value which will be reached by η when $R_2(t)$ reaches its maximum value in a zero period of time and remains set at this value.

In the still more special case in which R_u is constant and in which $R_2(t)$ is in the form given in (41) (P. 7.3.1.1), the expression (88) of the efficiency can be formulated beginning from relation (59.10):

$$\eta = \frac{W_u(\infty)}{W_o} = b\,(a-1)^{-b\frac{a-1}{a}}\,\gamma$$

This relation is expressed in the following form as a function of a and b:

$$\eta = (a-1)^{-b\frac{a-1}{a}} \Bigg[\frac{2}{(\frac{b}{a}+1)(\frac{b}{a}+2)}\, a^{b\frac{a-1}{a}-1} - \frac{2\,b}{(\frac{b}{a}+1)(\frac{b}{a}+2)(\frac{b}{a}+3)}\, a^{b\frac{a-1}{a}-3} (b\frac{a-1}{a}-2)$$

$$+ \frac{2\,b}{(\frac{b}{a}+2)(\frac{b}{a}+3)(\frac{b}{a}+4)}\, a^{b\frac{a-1}{a}-4}\, \frac{(b\frac{a-1}{a}-2)(b\frac{a-1}{a}-3)}{2\,!} - \dots \Bigg]$$

Wi $b \to 0$, a situation occurring, for example, in the case of $k \to \infty$ in relation (59), the efficiency η expressed above tends to the value:

$$\lim \eta_{b \to 0} = a^{-1} = \frac{1}{1 + \dfrac{R_u}{R_m}}$$

resulting from the fact that all of the bracketed terms, except for the first one, tend in this case to zero.

Practically, since the value of a is close to unity, expression (89.2) approaches very quickly to its boundary (89.3) as soon as b is smal with respect to unity.

When the impedance of the connections is not negligible, more particularly when the connections show a resistance R_3 (Figure 18): /92

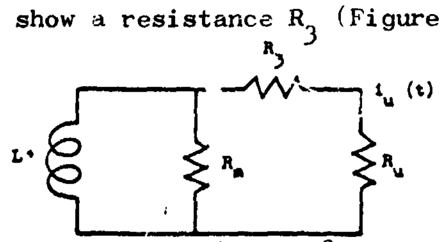


Figure 18.

the total energy dissipated by the connections is:

$$W_3 = \int_0^\infty R_3 \; i_u^2(t)\, dt = R_3 \int_0^\infty i_u^2(t)\, dt \qquad (89.4)$$

the total energy transferred in use is:

$$W_u = \int_0^\infty R_u \; i_u^2(t)\, dt = R_u \int_0^\infty i_u^2(t)\, dt \qquad (89.5)$$

It, therefore, follows that: /93

- the ratio between energy dissipated in the connections and that which is transferred in use:

$$\frac{W_3}{W_u} = \frac{R_3}{R_u} \qquad (89.6)$$

- the ratio between energy transferred in use and that which is transferred in the whole of the circuit made up by the use in series with its connections:

$$\frac{W_u}{W_u + W_3} = \frac{R_u}{R_u + R_3} \qquad (89.7)$$

When R_u is replaced by $R_u + R_3$ in expression (89), there is obtained a new expression representing the ratio between the total energy released into the circuit made up by the use in series with its connections and the initially stored energy. When this new expression is multiplied by the right side of expression (89.7), there is then obtained the ratio between the energy transferred in use and the initially stored energy, i.e. the efficiency of energy transfer:

$$\eta = \frac{R_u}{R_u + R_3} \; \frac{1}{1 + \dfrac{\int_0^\infty \frac{v^2(t)}{R_2(t)}\, dt}{\dfrac{\int_0^\infty v^2(t)\, dt}{R_u + R_3}}} < \frac{R_u \; R_m}{(R_u + R_3)(R_u + R_3 + R_m)} \qquad (90)$$

In order for this maximum limit of efficiency to remain high, it can be seen that R_3 should remain low with respect to R_u. /94

Bv means of these precautions, the case of an essentially dissipative use impedance is especially favorable to the production of efficiencies close to unity. Efficiencies such as this have been currently produced in laboratory experiments [55].

It should be noted that the removal of magnetic circuits has allowed obviating all of the losses which are characteristic of them.

Likewise expression (89.1) becomes:

$$\eta = \frac{R_u}{R_u + R_3} b' (a'-1)^{-b' \frac{a'-1}{a'}} \gamma' \tag{90.1}$$

with symbols a', b', γ' representing respectively the expressions a, b, γ in which R_u is replaced by $R_u + R_3$.

When $b' \to 0$, which occurs, for example, for $k \to \infty$ in relation (59), there may be found under these conditions:

$$\lim \eta_{b \to 0} = \frac{R_u}{R_u + R_3} a'^{-1} = \frac{R_u}{R_u + R_3} \quad \frac{R_m}{R_m + R_u + R_3} \tag{90.2}$$

Since for all practical purposes the value of a' is close to unity, expression (90.1) very quickly approaches its limit (90.2) as soon as b' is small with respect to unity.

7.4.2. <u>Coil use impedance</u>: /95

The energy transferred with use L_u is a magnetic energy whose value is provided by expression (67), the latter allowing us to express the efficiency:

$$\eta = \frac{W_u}{W_o} = \frac{L L_u}{(L + L_u)^2} \tag{91}$$

This efficiency is maximum when $L_u = L$ and it has the value in this case of 1/4. When this condition is realized, the relations (68) and (69) respectively provide:

$$W = \frac{W_o}{4}$$

$$W_2 = \frac{W_o}{2}$$

and, let us emphasize, this last energy W_2 is released in the form of heat.

In this case, only 25% of the initial magnetic energy is transferred into the use coil, 25% is retained in the storage coil and 50% is transformed into heat by Joule effect. All these values, let us recall (P. 7.3.2), are independent of the value of the resistance of the switch. /96

It can, therefore, be seen that with a purely coil use, the energy transfer efficiency is not only not favorable but even leads to an unavoidable and considerable degradation of energy.

7.5. The switch:

The switch (K_2 on Figures 8, 9, 14 and 15) should have the following characteristics:

When it is closed:

- It should allow passage of the total current.

- It should have, all connections and junctions included, a total resistance R_o sufficiently small so as to avoid introducing by dissipation of energy, an appreciable attenuation of the current trapped in the storage circuit L. Now, in such a circuit the trapped current is attenuated according to the rule:

$$I = I_o e^{-\frac{R_o}{L} t} \qquad (92)$$

When it is desired that the current does not drop below a value I (corresponding to a specific attenuation $\frac{I}{I}$) during a given storage period τ_s, this imposes for R_o the condition:

$$R_o \leqslant \frac{L}{\tau_s} \log \frac{I_o}{I} \qquad (93)$$

It is, therefore, necessary that R_o be sufficiently small. /97

By way of example, when the switch, including its junctions, is a superconductor when it is closed, this condition is met.

- It should have a sufficiently low coefficient of self induction with respect to the self induction of the storage circuit, as has been seen, in the case where the charge is carried out by means of an outside generator (P. 5.1).

During its opening:

When the use impedance is dissipative, it has been seen (relation (88) and (89)) that in order to produce good efficiency, it is required for the resistance of the switch to reach a relatively high value in a relatively short time with respect to the duration of discharge.

When the use impedance is coil in nature, it has been seen (P. 7.3.2) that the resistance of the switch has no effect on the efficiency but determines, on the other hand, the "rate" of discharge.

Insofar as concerns the coefficient of self induction of the switch, in addition to the conditions relating to the charging of energy (P. 5.1), it is possible to be persuaded either to maintain it below a value required by the need of not disturbing the discharge, or, on the contrary, to give it an optimum value which will allow producing a well specified law of discharge.

When it is open:

It should bear the voltage appearing at its terminals without giving rise to interfering phenomena such as flashovers, etc....

During its closure: /98

In general, the switch is not required to have closure times as short as those asked for with the opening. The swiftest closure times which could possible be used presently can correspond to commutations connected to cycles of the energy charging operation, such as we pointed out, for example, in P. 4.4.

Perfect switch:

In cases of transfer of active energy from the superconducting storage circuit into the use circuit, a switch which would dissipate practically negligible quantities of energy during the period of its use could be considered as perfect.

Several possibilities are then available for producing switches having such characteristics. We are going to examine the following ones:

- superconducting circuit component which is transited between superconducting and normal states;
- superconducting contacts which are opened and closed mechanically;
- conducting contacts which are opened and closed mechanically;
- destructible circuit component.

7.5.1. Component of the superconducting circuit which is transited:

A superconducting circuit component is used as switch [87] [93]. The switch is closed when the circuit component is in the superconducting state (zero resistance) and is "open" when it is in the normal state (non-zero resistance). The dynamics of such a switch (ratio of resistances in "open" and "closed" position) can be very high (cf. Table 2) in comparison with that of the best conventional switches used up until now in electrotechnics. Nevertheless, with presently available superconducting materials, it is not yet possible to produce, in absolute value, very high resistances in "open" position without using great quantities of superconducting material (P. 7.5.1.3).

Table 2. /99

Comparison between the resistivities becoming a factor in "open" position and "closed" position in the case of a superconducting switch and a conventional switch. These resistivities have only one indicative value. Indeed, the corresponding resistances of the switches produced should be compared. The conventional switches display in this case a dynamics which is much lower than that of superconducting switches.

Resistivities becoming a factor in	Superconducting switch		Conventional switch	
"Open" position	$10^{-4}\,\Omega \times cm$	Resistivity of the Nb-48% Ti in normal state	$10^{18}\,\Omega \times cm$	Resistivity in the mass of better-known insulators
			$10^{10}\,\Omega \times cm$	Possible surface resistivity
"Closed" position	$<3{,}6 \times 10^{-23}\,\Omega \times cm$	Upper known limit of resistivity in the superconducting state [96].	$1{,}7 \times 10^{-6}\,\Omega \times cm$	Resistivity of a conventional copper conductor
Order of magnitude of the possible dynamics: ratio of the resistivities becoming a factor in "open" and "closed" positions	$>2{,}7 \times 10^{18}$		$6 \times (10^{15}$ à $10^{23})$	

In spite of this, such a switch already has many advantages among which we can point out the following: /100

- no spark occurs during commutation;

- the electrical field can preserve a uniform distribution along the switch during and after its commutation. The switch can, in this case, sustain very high overvoltages without deterioration. This assumes that the energy dissipated, of which it is the origin, is sufficiently low, i.e.

$$W_2 = \int_0^{\infty} \frac{V_2^2(t)\,dt}{R_2(t)} \qquad (94)$$

During a discharge with a dissipative impedance, the same is true in the case where the resistance $R_2(t)$ of the switch reaches a high value with respect to the equivalent resistance of use in a sufficiently small period of time (P. 7.4.1) with respect to the duration of discharge,

- there are no moving mechanical parts.

The switch can be "activated" by acting (P. 1) either on the magnetic field to which the material is subjected, or on the current passing through it, or on its temperature, or on a combination of these quantities [55] [89]. In reality, at the present time, the action on the current passing through the switch has turned out to be the least advantageous owing to the electrical coupling which this introduces between the triggering circuit and the storage circuit as well as the difficulty of introducing on a practical basis the very high current pulses required. This difficulty turned out to be still more serious in the /101 special case considered in P. 7.5.1.3 in which the switch is formed by the storage circuit itself. To increase the current into the switch would then be the same as increasing the energy accumulated in the storage circuit. The best results have been obtained up until now by acting simultaneously on the temperature and magnetic field [55].

We have seen in P. 7.5 what were the chief qualities required of the switch. We are now going to see how they can be obtained.

7.5.1.1. Coil of the switch:

The value desired is produced by acting on the geometric arrangement of the superconducting material.

7.5.1.2. Transition velocity:

Very fast transition velocities have been produced by proceeding with two times [55] [89]. During the first time, the material forming the switch, in all its mass, is brought very close to transitional conditions. During the second time, these conditions at all points of the material are passed over very quickly by means of a wide amplitude pulse with a very steep rise front. Excellent results have been obtained by bringing the material in the first time very close to the critical temperature then subjecting it in the second time to a wide amplitude magnetic pulse (on the order, for example, of about 10 kilogauss) with a very fast rise front (on the order, for example, of about ten μs). In this way, there has been produced in the laboratory, using switches made up by Nb-Zr cables several tens of meters in length, total transitions from the superconducting state to the normal state in times on the order of the μs. Plans call, at the present time, for achieving much shorter transition times. Furthermore, it is well known, for example, that cryotrons [9] (although their transition is relatively easy to produce, since the mass of superconducting material is very small) can make the transition in time on the order of 10 s. It is known, on the other hand, that the time of relaxation of transition from the superconducting state to the normal state is less than 10^{-9} s. This allows, therefore, theoretical prediction of the capability for total transitions in times of this order of magnitude.

The act of proceeding in two times in order to trigger the opening of the switch also has other advantages. This allows, first of all when the switch is in its "closed" position, its use quite far from its critical conditions of transition. The result is that there is an improvement in stability and security of the circuit, for as will be seen (7.5.1.3) the switch cannot be "stabilized" by methods presently used to stabilize superconducting windings. Then, by acting on its temperature, it is possible to carry forward with precision the whole mass of the switch into conditions very close to the critical conditions of transition. Thus, it will then be possible to trigger the transition by means of a pulse with smaller amplitude while still allowing all points of the switch to amke the transition practically "at the same instant." Figures 19 to 21 specify this result.

Figure 19 diagrams as a function of time magnetic pulse B (t) assumed /103
picked off at a well defined point in space. The crosshatched strip depicts the area of corresponding values for which the different points of the switch make the transition (the lower part depicts the transition of the first point of the switch and the upper part that of the last point). The width ΔB of this area comes from the non-uniformity of the material or conditions in which is various points are found (parts of the switch located in regions of space where the field is relatively high with respect to the region where the field is depicted by the curve of Figure 19).

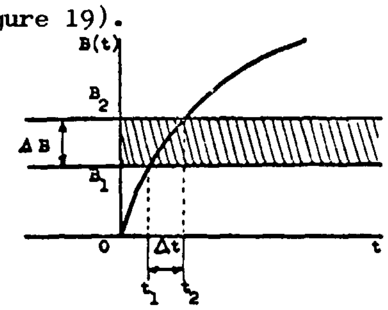


Figure 19.

To this range ΔB there corresponds a total duration of transition Δt which becomes proportionally greater as ΔB increases in width and as the slope of pulse B (t) becomes smaller.

It can, therefore, be seen that, in order to produce total transitions /104
in very short times, it is possible to:

- directly reduce range ΔB, for example, by using as uniform as possible a material for the switch which then carries the whole mass as uniformly as possible under conditions very close to critical conditions and by ensuring a spatial distribution as uniform as possible of the triggering magnetic pulse;
- increase the slope of the magnetic pulse; or again, in the case of a given pulse, to bring the switch to the limit of its critical conditions in such a manner as to shift area ΔB towards the origin of the coordinates on Figure 19. Indeed, it is in the vicinity of the origin, in this case, that magnetic pulse B (t) is maximum.

When it is desired to produce total and fast transitions, it is not possible to count on propagation from one transition begun at one point or at several points of the superconductor. Indeed, the propagation velocity of the transition into the superconductor is relatively low. Under better conditions it reaches several tens of meters per second [90] [91] and it is for this reason that it is found appropriate to initiate the transition as uniformly as possible, i.e. at all points of the switch "at the same time" [55] [89]. The resistance then appears abruptly and it "instantaneously" acquires the maximum possible value, i.e. that corresponding to the normal state at all points of the switch. Figure 20 diagrams the variation of resistance $R_2(t)$ of a switch when the /105 transition has not been uniformly initiated and Figure 21 diagrams the variation in resistance of the same switch when the transition has been uniformly initiated. In addition, on Figure 20, all points of the switch have not yet made the transition within the time interval shown.

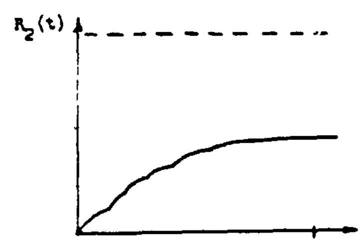


Figure 20.

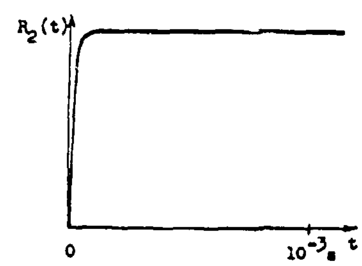


Figure 21.

We have used in P. 7.3.1.1, in order to depict the variation of resistance $R_2(t)$, the empirical function (41) whose general aspect is diagrammed by Figure 12. We are going to see what physical phenomena can reflect this law.

Let us consider the switch as made up by a uniform superconductor resulting from the longitudinal juxtaposition of a great number of identical domains having the following properties: /106

- the resistance of a domain in the superconducting state is zero;
- the resistance of a domain in the normal state has a finite value, such that if all the somains forming the switch are in the normal state, the resistance of the switch is equal to R_m;
- the transition from one domain is instantaneous with respect to the durations which are going to be considered.

At instant t, during transition, let us designate by x(t) the relative proportion of the domains which have not yet transited with respect to the total number of the domains forming the switch. It will follow that:

$$R_2(t) = R_m\,(1 - x(t)) \tag{95}$$

Let us assume that at any instant the ratio between the number of domains which transit per unit of time and the number of domains which have not yet transited is constant. Let us designate by k this constant ratio. It will follow that:

$$\frac{-\frac{dx}{dt}}{x(t)} = k \quad \text{with} \quad k > 0 \qquad (96)$$

By integration between instants 0 and t, the following is obtained:

$$x(t) = e^{-kt} \qquad (97)$$

and expression (95) becomes then

$$R_2(t) = R_m(1-e^{-kt}) \qquad (98)$$

which is the same as expression (41).

7.5.1.3. Resistance in "open" position: /107

Since the case in which it is desired that resistance in the "open" position be high is the most difficult to produce and since it is encountered when it is sought to produce fast discharges, we are going to examine it more particularly when the impedance of use is dissipative (Figures 8 and 9 in which Z_u is replaced by R_u). We have seen (P. 7.4.1) that in this case the values taken by the resistance of the switch have an effect on efficiency. In order for the energy to be transferred into the resistance of use R_u (and not dissipated into the switch), it is necessary for resistance $R_2(t)$ of the switch to become much greater than R_u within a time very short with respect to the duration of discharge.

7.5.1.3.1. Separate switch:

When the switch is distinct from the storage circuit and when it is such that it is not destroyed at time of discharge, it is "optimized" when, for a given total current I and a normal resistance R_m, the volume sl of the superconducting material used is minimum (s designating the section of the superconducting material forming the switch and l is length).

Designating the resistivity of the superconducting material in the normal state by ρ, the resistance of the switch in the normal state may be expressed:

$$R_m = \rho \frac{l}{s} = \rho \frac{sl}{s^2} \qquad (99)$$

and since /108

$$I = \delta_c s \qquad (100)$$

δ_c designating the critical current density, the following must be true:

$$sl = \frac{R_m I^2}{\rho \delta_c^2}$$

When R_m and I are given, it is therefore necessary, in order for sl to be minimum, that the following be true:

$$\boxed{\rho \delta_c^2 \text{ maximum}} \tag{102}$$

More particularly, this condition leads to the removal of the conductive sheathing which is generally used with superconducting materials in order to stabilize them. In reality, the conductive sheathing has the effect of reducing the apparent resistivity of the superconductor used in this way when it is in the normal state.

By acting on the configuration and geometrical arrangement of the material, it will be possible to increase the product $\rho\delta_c^2$. In addition, it will be found appropriate to select for the switch a material different from the one which will be "optimized" (cf, for example, relation (24)) for the storage winding.

We have undertaken in these last few years different works regarding these questions. Some works are chiefly oriented toward new materials as well as toward configurations with layers.

7.5.1.3.2. <u>Mixed switch</u>: /109

The solution described above requires the use of a specific quantity of superconducting material so as to produce the storage winding with self induction coefficient L which will have to preserve a negligible resistance during discharge chiefly in the case of Figure 8. Another quantity of superconducting material will have to be used in order to produce the switch with normal resistance R_m which will, on the other hand, have a negligible coil. It is more economical to cause the same superconducting material to simultaneously play the role of storage coil as well as switch [92]. In this case, then, the circuit shown in Figure 9 is used and it is the whole winding L (made up by an unsheathed superconductor) which has to be transited in order to "open" this storage circuit. This allows, for a given quantity of superconducting material, gaining the maximum possible normal resistance since it is made to transit the whole superconductor used. It is possible in this way to simultaneously profit from an improvement in efficiency as well as from a saving in material.

The optimization of the circuit can then lead to the making of a compromise between a condition of the type (102) corresponding, as it will be seen with a time constant characteristic of the minimum circuit and a condition of type (24) corresponding to a maximum of stored energy. At any rate, in both these types of relationships, it is possible to see the importance played by the critical current density. We shall not go further into details at this point.

7.5.1.3.3. Effect of the nature of the superconducting material: /110

We have seen (P. 7.4.1) in the case of discharge with a dissipative impedance that, in order to obtain a good efficiency of energy transfer, it was necessary for the resistance of the switch to reach, in a time sufficiently short with respect to the duration of discharge, a value R_m which was sufficiently high with respect to the values of use resistance. Let us assume, in the interests of simplification, that the resistance R_u is constant and that the preceding condition is realized. The value of induction coefficient L of the storage coil being given, the time constant $\frac{L}{R_m}$ characteristic of discharge of the circuit with itself is then small with respect to time constant $\frac{L}{R_u}$ of discharge of storage circuit with use. When the nature and quantity of superconducting material used are given, the time constant $\frac{L}{R_m}$ characteristic of discharge of the circuit with itself will clearly be minimum when R_m is made maximum. This will be accomplished when transit of the whole superconductor is made, i.e. when the storage coil itself is used as a switch (P. 7.5.1.3.2) which makes necessary recovery of the energy through the intermediary of a coupled circuit S (Figure 9).

Let us assume, again in the interests of simplification, a perfect coupling between windings L and S of the transformer made in this way. Let us designate by n_L and n_S the numbers of respective turns of these windings. There is then a strict relationship between L and S as follows:

$$S = L \left(\frac{n_S}{n_L}\right)^2 \qquad (103)$$

Under these conditions, the circuit of Figure 22 can be replaced by the equivalent circuit of Figure 23 in which resistance $R_2'(t)$ is equal to resistance $R_2(t)$, shown by coil L during its transition, multiplied by the square of the ratio of transformation $\frac{n_S}{n_L}$: /111

$$R_2'(t) = R_2(t) \left(\frac{n_S}{n_L}\right)^2 \qquad (104)$$

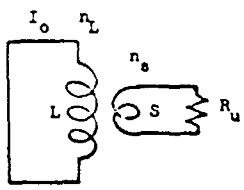


Figure 22.

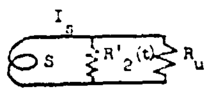


Figure 23.

At the initial instant, resistance $R_2'(t)$ is therefore strictly zero. The equivalent current I_S trapped in the equivalent coil S of the equivalent circuit of Figure 23 is equal to

$$I_S = I_o \frac{n_L}{n_S} \tag{105}$$

in which I is the initial current trapped in the real circuit (Figure 22).

It is confirmed that the energy thus initially stored in the equivalent circuit (Figure 23)

$$W = \frac{1}{2} \cdot S.\, I_S^2 = \frac{1}{2} \cdot L \left(\frac{n_S}{n_L}\right)^2. \left(I_o \frac{n_L}{n_S}\right)^2 = \frac{1}{2} \cdot L.\, I_o^2 \tag{106}$$ /112

is identically equal to the energy initially stored in the real circuit.

When coil L will transit, its resistance $R_2(t)$ will reach a value R_m and the equivalent resistance $R_2'(t)$ in the equivalent circuit will reach the corresponding value R_m' provided by (70):

$$R_m' = R_m \left(\frac{n_S}{n_L}\right)^2 \tag{107}$$

It is confirmed that the time constant characteristic of the equivalent circuit:

$$\frac{S}{R_m'} = \frac{L \left(\frac{n_S}{n_L}\right)^2}{R_m \left(\frac{n_S}{n_L}\right)^2} = \frac{L}{R_m} \tag{108}$$

is the same as the one for the real circuit.

It is likewise confirmed that the discharge time constant of the equivalent circuit:

$$\frac{S}{R_u} = \frac{L \left(\frac{n_S}{n_L}\right)^2}{R_u} = \frac{L}{\left(\frac{n_L}{n_S}\right)^2 R_u} \tag{109}$$

is the same as the one for the real circuit.

When the value of L is specified on a preliminary basis, the fact of wishing to supply a given use resistance R_u with a discharge time constant $\frac{S}{R_u}$ prescribes the value of S, i.e. the value of transformation ratio $\frac{n_L}{n_S}$. And in order for the efficiency to remain high, it is necessary for the characteristic /113 time constant $\frac{S}{R_m'}$ to be much less than discharge time constant $\frac{S}{R_u}$. This defines the value of R'_m which should satisfy the relation:

$$R'_m \gg R_u \tag{110}$$

which according to (107), defines the value of R_m:

$$R_m \gg \left(\frac{n_L}{n_S}\right)^2 R_u \tag{111}$$

<u>Noteworthy property</u>:

We are going to show that time constant $\frac{L}{R_m}$ characteristic of the circuit is always the same no matter how the superconducting material is used or arranged, and regardless of the value of the coil produced (fine wire and great number of turns, cable with heavy section and small number of turns, or even a massive circuit made up by a single turn [74a]), when the quantity of material used is specified, once a selection has been made of shape and bulk dimensions of the storage coil. This time constant is a function, on the other hand, of the nature of the material used, i.e. of its normal resistivity.

Let us assume a coil L produced using a length l of superconducting wire with section s. Designating by ρ the normal resistivity of the superconductor, the corresponding normal resistance will be:

$$R_m = \rho \frac{\ell}{s} \tag{112}$$

By keeping constant the quantity of material used, let us multiply by N (greater or less than 1) the number of turns in the winding. The length of wire used will be multiplied by N and the section of wire divided by N, in order for the total volume of the material: /114

$$N\ell . \frac{s}{N} = \ell . s \tag{113}$$

to remain constant. The normal resistance will then be equal to:

$$\mathcal{R}_m = \rho \frac{N\ell}{\frac{s}{N}} = N^2 R_m \tag{114}$$

Now, according to (22), the induction coefficient of this new winding will be:

$$\mathcal{L} = N^2 L \tag{115}$$

The characteristic time constant of the new circuit will be:

$$\frac{\mathcal{L}}{\mathcal{R}_m} = \frac{N^2 L}{N^2 R_m} = \frac{L}{R_m} \tag{116}$$

It is, therefore, independent of the value of the coil produced.

On the other hand, when the superconducting material is replaced by a material having a normal resistivity ρ times higher, the normal resistances R_m and R'_m will each be multiplied by ρ. The characteristic time constant of the circuit (relation (116)) will then be divided by ρ.

The result is that when, with a given quantity of a given material, the maximum possible stored energy is produced, the maximum "velocity" at which it could be hoped to discharge this energy will be limited by the normal resistivity of the material considered. This result has led us to direct our research /115 efforts toward the study of materials showing high resistivity in the normal state.

At any rate, since a material is given, it is always possible to reduce the characteristic time constant $\frac{L}{R_m}$ of the circuit, i.e. increase the maximum "velocity" of discharge possible. It is enough to produce, with the given quantity of material, a total coil less than the maximum coil which could be produced. This is what happens, for example, when a coilless switch (Figure 8) is added to a superconducting coil. It is always possible to increase the value of the normal resistance R_m of the switch beyond a given value. However, this is the same thing, for a determined stored energy, as using a quantity of superconducting material which is just as high.

7.5.1.3.4. Effect of the geometrical dimensions of the circuit:

It can be asked how the characteristic time constants and stored energies vary in the case of geometrically alike windings. According to the results of the preceding paragraph, the law of variation will be the same no matter what may be the values of the self induction coefficients of the windings considered. We are therefore going to consider two strictly similar windings, i.e. such that all dimensions of one winding (including therein the wire dimensions used) are a multiple α of the corresponding dimensions of the other.

Let us designate by:

L and L' ... the coefficient of self induction of the two windings,

R and R' ... their normal resistances,

I and I' ... the trapped currents,

H and H' ... the magnetic fields at homologous points, /116

W and W' ... the corresponding stored energies.

By using the degrees of homogeneity of these different quantities with respect to length, it follows that:

$$L' = \alpha L \tag{117}$$

$$R' = \frac{R}{\alpha} \tag{118}$$

and hen the same current density is preserved:

$$I' = \alpha^2 I \tag{119}$$

The relationship between the characteristic time constants of the two windings is deduced from it:

$$\frac{L'}{R'} = \alpha^2 \frac{L}{R} \tag{120}$$

and the relationship between the stored energies:

$$W' = \frac{1}{2} L' I'^2 = \alpha^5 \frac{1}{2} L I^2 = \alpha^5 W \tag{121}$$

Indeed, since the superconducting material used is the same for both windings, its critical field will be the same in both cases for a given current density. In this case, it will be appropriate to compare the energies stored for a same field in the vicinity of the material.

Since:

$$H' = \frac{I'}{I} \frac{H}{\alpha} \tag{122}$$

it will be appropriate in order to preserve the same field at homologous points to use a current I" given by the relation:

$$I" = \alpha I \tag{123}$$

In this case, in order to use the minimum of superconducting material, it will be appropriate to preserve the same current density in both windings. This will only be possible when the thickness of the second winding is not multiplied by α. However, in this case, both windings will no longer be geometrically alike, since the thicknesses of the windings have been kept identical. /117

The resistances and coefficients of self induction (for windings with narrow thickness) will be connected by relations:

$$R" = \frac{R}{\alpha} \alpha = R \tag{124}$$

$$L" = \alpha L \tag{125}$$

whence is deduced: the relation between characteristic time constants:

$$\frac{L"}{R"} = \alpha \frac{L}{R} \tag{126}$$

the relation between stored energies

$$W'' = \frac{1}{2} L'' I''^2 = \alpha^3 \frac{1}{2} L I^2 = \alpha^3 W \tag{127}$$

In this case, the stored energy will increase as α^2 (with the winding thickness remaining constant). Relation (24) $W \sim m^{3/2}$ with constant current density and induction may be recovered.

The characteristic time constant of the circuit increases as α whereas the stored energy increases as α^3.

In order to conclude discussion of the special features of the switch formed by one circuit component which Φ is made to transit, it can be said that with this type of switch it is possible to obtain very fast openings but that with presently available superconducting materials the values of the final resistance (in open position) are not now very high. /118

7.5.2. <u>Mechanically actuated superconducting contacts</u>:

The switch is formed by superconducting contacts which are mechanically separated in order to cause opening. This opening is not as swift as in the preceding case and, in addition, it can cause arcing.

This disadvantage is partially remedied by using contacts under vacuum. It is even possible in order to aid extinction of the arc, to install a capacitor at the switch terminals [94]. It is additionally possible to design the switch in such a manner that the parts which are separated last during opening, or those coming in contact first during closure, are duplicated by a superconducting contact forming a sort of lock in closed position. In this way, the components intended for the commutation function are separated from those used in the closed position for ensuring continuity of the superconducting circuit.

The advantage of such a switch is such that it can have practically zero resistance in closed position and practically infinite resistance in open position.

The possible uses of such a switch are as follows:

- it can be used for discharge of a stored energy with a dissipative impedance of use (switch K_2 on Figures 8 and 9). It is necessary that, during its opening, its resistance R_2 (t) passes on abruptly (time period Δt on Figure 24) /119 with respect to the duration of discharge, from a very low value to a very high value with respect to the values of use resistance $R_u(t)$. As long as the resistance shown by the switch remains quite small, the time constant of the storage circuit remains quite large and the energy remains stored.

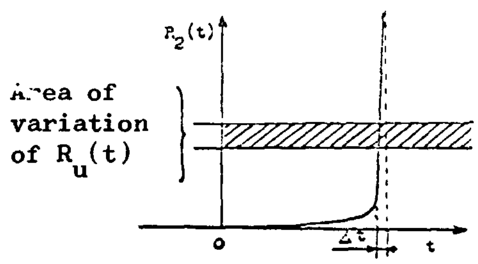


Figure 24.

As soon as the resistance of the switch becomes higher than the use resistance, the stored energy is discharged into use;

- it is possible to use this switch as a safety device intended to shunt in another switch (P. 6.3). In this case, it can be actuated with practically no difference in potential appearing between its terminals, i.e. without causing an arc;

- it can be used as a disconnecting switch. It is only actuated when the current passing through it is zero. Two examples of this have been associated with the static transformer, forming the subject of reference [70], used for energy charging. In this case, the voltage at its terminals is zero when it is actuated. No arc can appear; /120

- it can even be used as an isolating switch in order to separate two superconducting circuits which are to be isolated from each other and between which high voltages can appear. This case can occur, for example, when it is desired to separate the energy charging device from the storage device before carrying out the discharge. The switch can then sustain in the open position very high differences in potential which are a function of the distance between its contacts.

7.5.3. Mechanically actuated conducting contacts:

Some switches - isolating switches, circuit breakers - can have in their closed position contact resistances which are sufficiently small to be able to be used in place of the preceding switches. Their contacts and connections can be made at ambient temperature, or brought to a low temperature in order to reduce their resistance.

7.5.4. Destructible circuit component:

The switch can be formed by a circuit component (whether superconducting or not) which is destroyed. The circuit should be such that, during rupture, the electrical arc coming from the destruction of the material is interrupted quickly enough. Such a switch can be used under the same conditions as the two preceding ones although, for practical purposes, it only operates at the opening. In addition, the destroyed part must be replaced following each operation.

7.6. Special features of the circuits: /121

The circuits should have characteristics different from those which are sought after in the technology of intense magnetic fields. Among others:

Since windings are the seat of large flux variations during discharge, there can appear high voltages as well as substantial induced currents in the adjacent conducting masses. Consequently, the superconducting turns (if it involves a winding formed from turns) should be electrically insulated from each other. Likewise, when the winding is formed by several layers or by several windings (primary, secondary windings...etc.) the layers or windings should be suitably insulated.

The use of metal carcasses or metal hoops which could behave as short circuit turns coupled to the storage circuit and could be the location of large Foucault currents is rejected out of hand. Likewise, there are precautions to be taken, insofar as concerns the cryostat enclosing the winding, when it is metal or when it includes metal or metallized parts. It can be appropriate either to use materials with high electrical resistivity, suitably arranged so as to reduce to the minimum the coupling with storage device, or to use electrically insulating materials. This, therefore, more particularly prohibits some of the "stabilization" methods presently used to produce permanent magnetic fields.

Magnetic materials now on the market are all found to be saturated in the vicinity of 25 kG. It has, for this reason, been found generally advantageous not to use a magnetic circuit. This then compels precautions to be taken so as to obtain a good magnetic coupling between windings (e.g., primary and secondary). Success is achieved in the latter by using windings which have a large diameter with respect to the thickness of windings. Sufficiently insulated intermeshed windings can also be used. /122

In the case of high-speed discharges, it can even be appropriate to take special precautions for reduction of the "skin effect" on the conductors.

8. Some Presently Possible Applications /125

Since the use of superconductors for storage and discharge of electrical energy is quite new, there are as yet no industrial designs. Nevertheless, the possibilities are many, ranging from the mere substitution for traditional energy discharge devices to new uses of energy which the special properties of superconductors now make possible.

Specifically, let us recall that superconductors allow:

- storage of very high energies (P. 6.7), thus extending the present-day applications of conventional devices for storage of electrical energy;
- producing very high densities of stored electrical energy (P. 6.7), leading to reduced space requirements;
- producing very fast discharges and consequently high powers;
- supplying use impedances with very different values, since several use circuits can even be supplied at the same time by means of conventional secondary windings (P. 7.2);
- producing very high currents or voltages (P. 7.2);
- obtaining high efficiencies since the superconducting device offers new capabilities for adaptation of impedance and in this way producing discharges damped with the dissipative impedances of use (P. 7.3.1);
- and, lastly, being able to store energy at very low temperatures.

In the following it will be possible to discriminate, in the passing, between:

- fast discharges when it is chiefly sought to produce either very short release times or great powers;
- "slow" discharges when the chief interest is in the possible duration of use of energy which has been stored.

In the future, the combined use of both these types of discharges can be /126
planned on in some cases.

8.1 Discharges into gas:

The discharge into a gas causes the creation of a plasma consequent to the ionization of gas molecules. Depending on the initial pressure of the gas it is possible to obtain different types of discharges [98]. The energy and degree of ionization of the particles forming the plasma are connected to the electrical field in the discharge. The number of these particles is chiefly connected to the current. In discharge, the ionized particles are in motion and their spatial distribution can gradually develop in the course of time. This gradual development can even lead to a deformation of the plasma (the phenomenon of contraction is one well-known example of this) or a motion of the whole plasma (this is, for example, the case during ejection of plasma by a plasma gun (P. 8.1.5)). The plasma can be considered, taken as a whole, as a deformable circuit leading, in its deformation, to a modification in the spatial distribution of the magnetic field. The resilt is that there is a variation of flux and the corresponding back electromotive force is given by the term $\frac{d\,L_u(t)}{dt}\,i\,(t)$ of equation (86). To this back electromotive force there corresponds an energy expressed by the term $\frac{d\,L_u(t)}{dt}\,i^2(t)\,dt$ of equation (87) which, as seen above (P. 7.3.4) is distributed in equal quantities as magnetic energy and work.

8.1.1. <u>Practical production of power supply</u>:

8.1.1.1. <u>Discharges between electrodes</u>:

It is possible to connect the electrodes, among which it is desired to perform the discharge, directly to terminals A and B of the storage circuit as shown on Figure 25.

/127

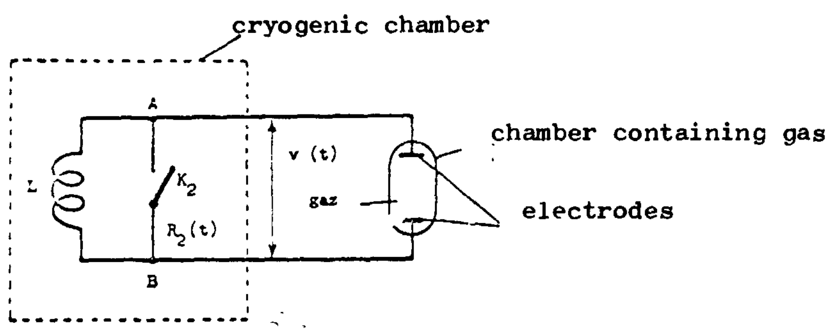


Figure 25.

Since the superconducting coil L has been charged by a direct current I_o, when the switch K_2 is opened, there appears between points A and B a difference in potential v(t) given by relation (29) (in which it is enough to replace R(t) by resistance $R_2(t)$ of the switch) as long as the discharge into the gas has not begun:

$$v\,(t) = R_2\,(t)\,I_o\;e^{-\frac{1}{L}\int_0^t R_2(t)\,dt} \tag{128}$$

It has been seen (P. 7.3.1) that this difference in potential is increasing (up to a maximum). Consequently, as soon as the disruptive voltage is reached in the gas (P. 9), discharge begins into the gas.

The connections of the electrodes at points A and B are made by means of normal conductors (e.g., copper or aluminum). These conductors can form a line, e.g., a coaxial line if required, etc.... In order to avoid during discharge the disadvantages arising from an accidental transition of the superconducting storage coil, it is possible to sheathe the superconductor forming it with a normal conductor (e.g., aluminum or copper) to which are then attached the connections of the discharge electrodes [87]. It is also possible to supply these electrodes by means of a secondary winding formed by a normal conductor and coupled by induction to the storage circuit (P. 7.2). /128

When it is desired to lower the initial discharge voltage, it is possible to ionize the gas on a preliminary basis by using any one of the well-known conventional methods. It is also possible, by using several secondary circuits coupled to the storage circuit, to clearly separate the functions:

- beginning of discharge, requiring a high voltage but little energy;
- power supply with energy of discharge in its true sense, requiring high current.

Figure 26 provides an example of such a design: /129

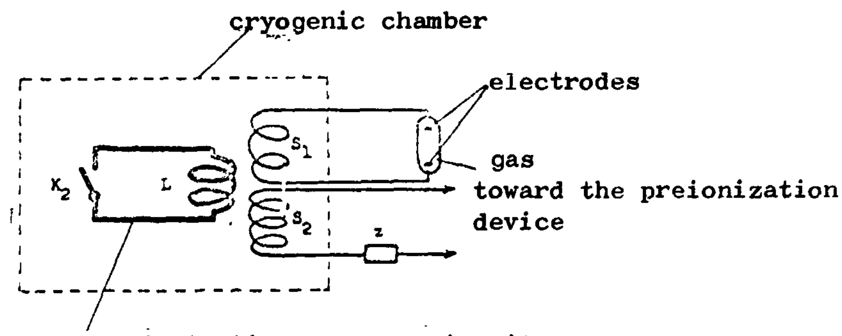


Figure 26.

The secondary winding S_2 is exclusively intended to supply the initial voltage required to supply with power the gas preionization device. Depending on the method used (not shown), it is possible to make a series arrangement with winding S_2 with an impedance z (which can be a resistance, a capacitance, etc.) in such a manner as to limit the power delivered by this circuit. On the contrary, winding S_1 is provided in order to power the discharge by transferring to it, once the preionization is carried out, the greater part of the initially stored energy. In the case of Figure 26, it is possible to use the superconducting coil L (P. 7.5.1.3.2) directly as discharge switch.

8.1.1.2. Discharges by induction: /130

It is possible to initiate and power the discharge into the gas exclusively by electromagnetic induction without it being necessary to use electrodes. To do this the gas, contained in an isolating chamber, must be handled in such a manner that once ionized, i.e. when it becomes a conductor, it forms itself a conducting circuit coupled by induction to the storage circuit (P. 7.2).

The ionization of the gas can be started directly by the electrical field which appears during opening of the storage circuit, or it can even be caused by an independent external means or itself powered by a supplemental secondary circuit such as S_2 on Figure 26.

8.1.2. Use for research on controlled nuclear fusion:

Up until now, and practically everywhere, the "machines" used for research on controlled nuclear fusion [99] [100] [101] were powered with energy beginning from batteries of capacitors. In spite of the serious disadvantages which we shall review, capacitors have formed the only state-of-the-art means available for supplying energy to experiments carried out in laboratories throughout the world during the sixteen years of research which have just elapsed since the inauguration of the first large scale program (the Sherwood project, in 1951 in the U.S.A.).

On one hand, the energies which capacitors could store were very limited. Several megajoules was still regarded as a limit for practical purposes. The space requirement (in part responsible for stray impedances in the case of high-speed discharges)and cost of capacitors are already considerable just for energies amounting to several magajoules and increase proportionally to the stored energy [54]. It is hard to imagine increasing their number by any large factor. The storage of much higher energies in capacitors in the absence of any considerable progress made in their technology does not therefore appear feasible in the near future. /131

On the other hand, the efficiencies of transfer of energy to the plasma were very low. Indeed, the discharges produced were oscillating and only were damped in a limited number of periods, sometimes compelling use of circuits of the "crowbar" type in order to avoid the disturbances this situation caused in the plasma. These oscillations were caused by the fact that the extra coil of the capacitors and their connections to the plasma was not negligible with respect to the large capacities (measured in microfarads) required to store the energy.

The goal pursued is to "heat" the plasma, i.e. to cause it to absorb a degree of active electrical power. Now, precisely these unavoidable oscillations are characteristic of a poor absorption of energy by the plasma, i.e. a poor efficiency of transfer of stored energy to the plasma. For this reason, research on controlled fusion finds itself in an impasse in the matter of supplying large quantities of energy to the plasma.

At the present time various projects are under study to replace batteries of capacitors by electromechanical generators capable of converting into electrical energy the kinetic energy stored in a spinning flywheel [102] [103]. These machines can only release energy at a velocity limited by the maximum /132 corresponding acceleration compatible with their mechanical resistance. They would be used to charge coils whose energy, thus stored, world then be discharged more rapidly into the plasma by means of suitable switches. The switch represents, in this case, a critical point in the project.

Fortunately, we have seen (P. 8) that superconductors allow, in relatively reduced volumes, the direct storage, then release, of very high energies (P. 6.7), greater by several orders of magnitude to those stored up until now with capacitors. The space requirement and cost of these superconducting devices grow less quickly than the stored energy which makes their use even more advantageous when higher energies are to be stored [54]. They supply very fast discharges on a direct basis [55] (discharges such as the one diagrammed on Figure 11 are quite advantageous) as well as a better efficiency of transfer of energy to the plasma, since the extra coil of the connections does not introduce oscillations (P. 7.3.4 and 7.3.1) and since the order of magnitude of the stray capacities of the connections was too small. In addition, in the domain of controlled fusion, they open the field to new experiments which would not have been possible with customary energy storage devices [104].

8.1.3. Power supply of sources of neutrons:

There are many types of sources of neutrons. We are concerned here with those involving fusion reactions either in a plasma or by bombardment of a target by means of accelerated particles. It has been found to be possible at the present time, on the one hand, to supply by means of very small sized superconducting circuits those sources until now supplied by capacitors, on the other hand, to plan on a power supply from more intense sources.

8.1.4. Power supply of "flash" tubes: /133

8.1.4.1. Laser pumping:

It is now possible to power, using superconductors, those "flash" tubes used to pump lasers.

This allows:

- reduction of the space requirement for energy sources powering present lasers;

- planning for much greater power supplies for lasers.

Such experiments have already been carried out [105] successfully at low power by using energies successively of 500 joules and 2 kilojoules. The circuits used correspond respectively:

- to the diagram of Figure 25 for tests without preionization of the gas;

- to the diagram of Figure 27 for tests with a preionization produced beginning from an external circuit: a previously charged capacitor C kept at a continuous potential, high with respect to the electrodes of discharges, the triggering electrode of the tube (formed by some mesh of a thin metal wire on the outside of the tube);

- to the diagram of Figure 28 or to that of Figure 26 for the tests with preionization obtained directly beginning from the storage circuit.

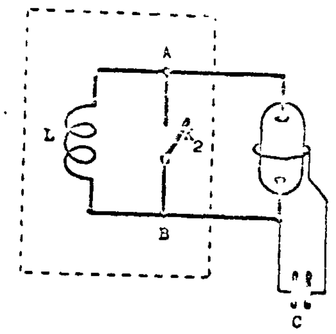


Figure 27.

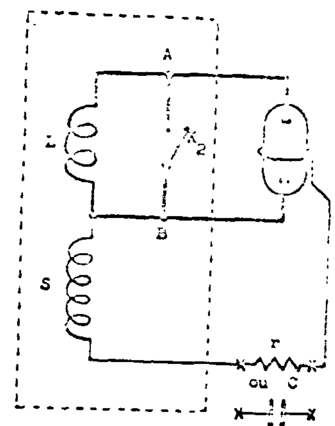


Figure 28. /134

In these circuits only the circuit formed by winding L and switch K_2 can be superconducting. The other windings are made up by normal conductors. Paragraph 9 describes an example of determination of such a power supply.

The superconductors are well adapted to production of devices in new configurations for supplying power to "flash" tubes and pumping lasers [104].

8.1.4.2. Power supply of flash generating tubes for photography: /135

Under the same conditions as above, it is possible to consider supplying luminous flash generating tubes with considerable power for use in photography.

8.1.4.3. Power supply of x-ray generating tubes:

Plans can even be made for supplying x-ray generating tubes by means of very high voltage pulses delivered by superconducting devices.

8.1.5. Power supply for plasma guns:

The plasma gun allows ejection of ions at very high velocities. These ions can be produced directly by ionization of the gas supplying the gun. Such a gun can in this case turn out to be extremely advantageous for space propulsion. Indeed, there could be produced here velocities of matter ejection as high as possible so as to collect, beginning from the ejected matter, the maximum quantity of motion which ultimately allows embarking at the beginning the minimum mass of matter intended for use as a projectile.

Up until the present time, the only source of energy possible for the creation and propulsion of these ions was represented by capacitors. Nevertheless, their weight and volume removed from this type of gun its chief advantage: its lightness. From now on, plans can be made for replacing these capacitors by superconducting sources which are much lighter, much less bulky and more powerful. The maintenance of low temperatures required does not raise any problems of principle since at the present time low-power microrefrigerators are already in use on board satellites in order to keep electronic circuits at very low temperatures.

8.2. Discharges into liquids: /136

Power supply for testing dielectric strengths

The very high voltage pulses that can be produced by means of superconducting devices can be used for study of the dielectric strength of insulating liquids.

8.3. Discharges into solids:

8.3.1. Power supply for testing dielectric strengths:

Tests of dielectric strengths of solids can be powered under the same conditions as above.

8.3.2. Explosions of wires:

Much research is presently being carried on concerning explosions of wires [106] and has already been responsible for several applications. The equipments used are exclusively powered by means of capacitors. It is now possible to substitute superconductors for them.

Among the applications of explosions of wires we shall mention the following:

8.3.2.1. Creation of shock waves:

The explosion of the wire in an equipment environment creates a shock wave which can be used either to carry out mechanical deep-drawing (explosion in water or in oil) or to trigger chemical reactions (wire detonators), etc.

8.3.2.2. Ignition of high power arcs: /137

Some wind tunnels used for aeronautical research are powered by discharge of electrical energy into a gas under pressure. The durations of discharge are on the order of a fraction of a second. Now, Paschen's law (cf., for example, [98]) shows that in order to start an electrical discharge in a gas under pressure there is required an initial voltage which rises with increase in gas pressure.

In order to avoid use of too high initial voltages, the gas is then preionized by causing explosion of a conducting wire previously connected between the two discharge electrodes.

Such a device could very simply operate beginning from a superconducting power supply (for example, according to a diagram of the type shown in Figure 29). In this case, it could be planned on using considerable energies with respect to those available up until now.

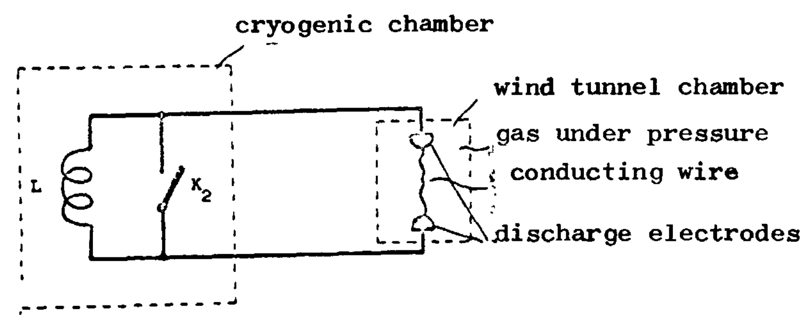


Figure 29.

8.3.2.3. Metallizations: /138

Some metallizations are carried out by explosion of a metal wire. The required electrical energy could be supplied by a superconducting device.

8.3.3. Spot welding:

The electrodes of some spot welding apparatus are powered by discharge from capacitors. It is possible to plan, using superconductors, on reaching spot welding powers much greater than those produced heretofore.

8.3.4. Power supply of cryogenic circuits:

In low-temperature devices where there is a requirement for using electrical energy, plans can be made for replacing the capacitors or conventional electrochemical batteries by superconducting devices built into the cold part. It is possible in this way to avoid introducing electrical connections between the cold and hot parts of the apparatus. On the other hand, it can be easier to introduce from the outside the required energy into the superconducting storage device (P. 4).

8.4. Power supply of other circuits:

8.4.1. Superconducting memories:

We shall not discuss in length this well-known application of superconductors which consists in replacing the magnetic memories of the computers by microcircuits for storage and discharge of energy, used to maintain in the normal state or superconducting state those superconducting transition microswitches designated under the general term of crytrons [6]. The energies becoming a /139 factor in such circuits are extremely small. They can, for example, be less than 10^{-10} joules per commutation.

8.4.2. Power supply of particle accelerators:

At the present time, in order to power, for example, the electromagnets of the proton synchrotrons, it is required to commutate on a periodic basis energies on the order of several megajoules with periods on the order of a second. For this, use is currently made of rotating machines which store energy in kinetic form during the half-period in which the energy is to be taken from the electromagnet. However, these machines have a limited endurance [107].

P. E. Smith [107], after having considered replacing such machines by supercorducting devices, found that for energies on the order of 1000 megajoules a superconducting system would be more economically viable than any other system presently in use or planned (this finding clearly will not surprise us [54]) and he suggested a storage system possible for an accelerator designed for 300 GeV whose peak value of accelerating field could put into play an energy of 300 megajoules.

8.4.3. Electrical power supply of rockets:

Superconducting devices are planned [108] [109] so as to ensure power supply of electrical circuits of rockets during flight.

8.4.4. Electrical power supply of vehicles: /140

The advantage of powering vehicles such as passenger automobiles, beginning from electrical energy stored in superconducting circuits playing the role of batteries, has been emphasized [110].

8.4.5. Storage of reactive energy:

At the present time research is being carried out by a few organizations on storage of reactive energy in the networks of transportation and distribution of electrical energy. However, involved here is a problem which is quite different from those which we have examined.

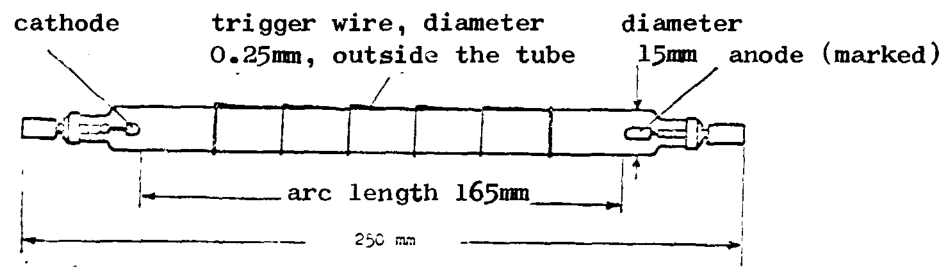
9. Examples of Simplified Predeterminations of Storage and Energy Discharge Systems /143

9.1. Power supply of a laser flash tube:

The characteristics of the tube to be powered, which has been selected as an example, are provided in paragraph 9.1.1 and the determination of the corresponding superconducting power supply is carried out in the paragraphs below:

9.1.1. Characteristics of the tube to be powered:

It is a matter of powering a type F X 47 flash tube constructed by E.G. and G. (Edgerton, Germeshausen and Grier, U.S.A.) for pumping lasers and formed by two electrodes located in a Xenon environment under a transparent quartz cover. Diagram and dimensions of the tube are given by Figure 30.



F X 47 flash tube

Figure 30.

9.1.1.1. Data from the designer: /144

The performances and operating conditions provided by the designer for a capacitor powered tube are as follows:

9.1.1.1.1. Performances:

Maximum power supply energy: 5000 joules

Luminous corresponding energy: 20,000 candles x second

Duration of flash (estimated at 1/3 of the height of pulse):
2.2 milliseconds

Standard repetition rate: 1 flash every 4 minutes

Nominal service life of the tube: 7,500 flashes.

9.1.1.1.2. Operating conditions:

The tube should be powered (Figure 31) beginning from a capacity of 2200 µF, charged with 2.15 kV at the maximum (5000 joules), in series with a suppressor choke of 0.85 mH and 0.044 Ω of resistance (able to endure a maximum peak current of 5000 amperes for a 1 ms pulse) intended to limit the peak current to a value without danger for the tube.

Initial voltage between discharge electrodes: 1000 volts.

Self-priming voltage: greater than 10 kV for a new tube, then dropping to 3 kV (it is possible to use the tube directly in this way by powering it by means of triggered spark gaps or relays).

Standard command voltage of the intitator: 25 kV.

9.1.1.2. Conventional electrical circuit: /145

The conventional electrical circuit for power supply of the tube is shown by Figure 31:

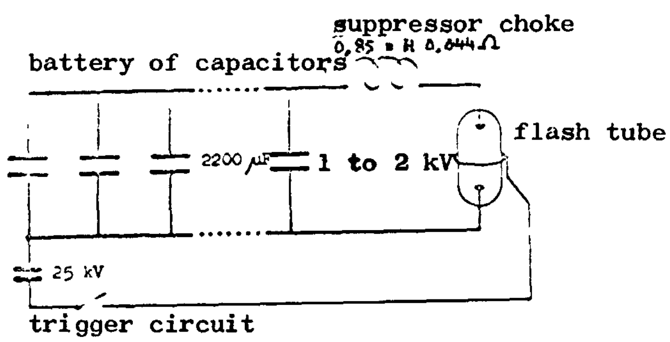


Figure 31.

In the special case considered, the weight and space requirement of the different circuit components are the following:

Battery of capacitors:

It is made up by 48 capacitors with 40 μF each, totaling a capacity of 1920 μF, installed on a rack.

Volume of one capacitor: 1 dm^3

Total volume of the rack: 250 dm^3 (high voltage safeties not included).

Total weight of the rack: 100 kg (high voltage safeties not included).

Suppressor choke: /146

Dimensions (including supports): 23 x 23 x 12 cm

Total volume: 13 dm^3

Weight of coil: 7.8 kg

Triggering circuit:

Total volume: a few dm^3

Charges:

The chargers used to charge the capacitors will not be examined.

9.1.1.3. Experimentally recorded data:

The designer's data, supplied in order to cause the tube to operate under well-determined conditions and beginn .g from capacitors, are not sufficient to allow definition of the power supply under conditions very different from those anticipated. It was consequently necessary to experimentally record the data lacking. This recording was performed by powering the tube beginning from its battery of capacitors and allowed specification of the following points:

- the nature of the impedance shown by the tube during discharge can be considered as a first approximation as chiefly dissipative (P. 7.3.1);
- the equivalent resistance recorded is shown as a function of time, for charging voltages of the 2 kV, 1.5 kV and 1 kV capacitors, by the curves of Figure 32. It is, for example, on the order of 0.4 Ω to 0.8 Ω.

/147

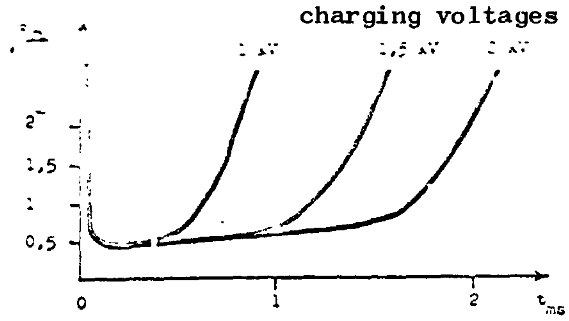


Figure 32.

Likewise, the curves depicting the resistance of the tube as a function of the current have been plotted. They show that, for a given current, the resistance of the tube is a function of the previous values of the current. Indeed, in order to simplify the calculations which will follow, it is possible to use the order of magnitude of the initial resistance of the tube when it is energized, since the greater part of the energy becomes a factor at the beginning of discharge (this only serves as a first approximation since the experiments carried out [105] by powering such tubes by superconducting devices show that indeed it is the order of magnitude of the discharge voltage that could be considered as constant. See the calculation corresponding to (P. 9.3)). Nevertheless, in the calculation which follows, the tube will be compared, when it is energized, to a pure resistance of 0.5 Ω.

9.1.2. Initial data of the calculation: /148

By taking into account the limitations and results discussed above, we are going, for example, to consider the operational case defined in Table 3.

The characteristic time constant τ_2 of the storage circuit is selected much lower than time constant τ of the discharge in order to allow a good efficiency of energy transfer (P. 7.4.1).

The rise time t_1 of the current was not selected shorter in order to limit on a preliminary basis the power of the shock wave in the tube.

Table 3.

Designation of the quantity	Symbol and given value	Relation to be used
Available energy	$W_u(\infty) = 2\,000$ joules	(56)
Time constant of discharge	$\tau = 4 \times 10^{-3}$ s.	(50)
Time constant characteristic of the storage circuit	$\tau_2 = 10^{-1}\tau$	(43)
Rise time of the current	$t_1 = 10^{-1}\tau$	(44)
Resistance of the tube	$R_u = 0.5\ \Omega$	

9.1.3. Determination of power supply characteristics:

In order to facilitate calculations we assume the impedance of connections to be negligible (P. 7.3.1.1).

It follows from (50):

$$L = 2 \times 10^{-3} \text{ henry} \qquad (129)$$ /149

It follows from (43) and (50):

$$R_m = 5\ \Omega \qquad (130)$$

By solving the transcendental equation (44) for K, it follows that:

$$K = 2.4 \times 10^{3}\ s^{-1}. \qquad (131)$$

By means of (59.10) it is possible to calculate the initial energy W_o which it is necessary to store in order for given energy $W_u(\infty)$ to be transferred between zero and infinite instants. The calculation provides successively:

according to (56):

$$a = 1.1 \tag{132}$$

according to (59):

$$b = 0.208 \tag{133}$$

according to (59.1):

$$\alpha = 1.9811 \tag{134}$$

according to (59.2):

$$\beta = 0.811 \tag{135}$$

according to (59.10):

$$W_u(\infty) = W_o \times 0{,}208 \times (0{,}1)^{-0{,}0189} (3{,}35 + 0{,}36 + 0{,}14 + 0{,}07 + ...) = W_o \times 0{,}85 \tag{136}$$ /150

The following is then true:

$$W_o = 2360 \text{ joules} \tag{137}$$

whence the current to be trapped, obtained beginning from (1)

$$I_o = 1540 \text{ amperes.} \tag{138}$$

We can again successively calculate: the value of the resistance of the switch at instant t_1 in which the current is maximum, according to (46):

$$R_2(t_1) = 3.05\ \Omega \tag{139}$$

the slope at the origin of function $R_2(t)$, according to (41):

$$\left(\frac{d\ R_2(t)}{dt}\right)_{t=0} = R_m k = 1{,}2 \times 10^4\ \Omega/s \tag{140}$$

the slope at the origin of the rise front of the current into R_u, according to (49):

$$\left(\frac{d\ i_u(t)}{dt}\right)_{t=0} = 3{,}7 \times 10^7\ \text{A/s} \tag{141}$$

Comment 1

In reality, before the discharge begins in the tube, only resistance $R_2(t)$ of the switch is connected to the terminals of the storage coil L. The voltage appearing at the terminals of the tube is then provided by relation (29) in which it is necessary to replace R(t) by the single resistance $R_2(t)$ provided by equation (41), using the numerical constants obtained with (130) and (131). /151

It follows in this case that:

$$v(t) = R_a I_o\ e^{\frac{1}{k\tau_2}} (1 - e^{-kt})\ e^{-\frac{1}{k\tau_2}(kt + e^{-kt})} \tag{142}$$

The derivative of this function with respect to t may be expressed:

$$\frac{dv(t)}{dt} = R_u I_o \, e^{\frac{1}{k\theta_2}} \, e^{-\frac{1}{\theta_2}\left(kt + e^{-kt}\right)} \left[k e^{-kt} - \frac{1}{\theta_2}\left(1 - e^{-kt}\right)^2 \right] \tag{143}$$

The factor out of brackets is positive and different from zero except for t infinite. The factor between brackets is reduced to zero at the same time changing sign for one value of t and one alone when t increases from zero to infinity. Consequently, the function v(t) is first of all increasing, passes through a maximum for the value of t which cancels the bracket which we have just mentioned, then decreasing. Furthermore, it is useless to recalculate the value of t corresponding to the maximum of v(t) since we have seen (P. 7.3. 1.1.1) that this value of t was independent of the constant value of R_u, which had been assumed infinite so long as the tube was not energized. Consequently, the maximum of v(t) (equation (142)) takes place for the value $t = t_1$ already provided by equation (44). (It can easily be confirmed with equation (143).) In reality, the circuit will have to be designed (P. 9.1.4) in such a way that the tube is energized before v(t) has reached its maximum. This will be done for two reasons:

- in order to avoid risking non-energization of the tube;
- in order to transfer maximum energy into the tube during discharge.

The numberical calculation of the maximum value of v(t), produced by carrying into equation (142) the value of t_1 given in Table 3, provides às follows:

$$v(t_1) = 3340 \text{ volts.} \tag{144}$$

(It is also confirmed that this voltage is less at the upper terminal than would be produced by multiplying the value $R_2(t_1)$, given by (139), by the current I assumed constant and equal to I_o. It follows that: $I_o R_2(t_1) =$ 4630 volts.) /152

As soon as the discharge into the tube has begun, the relation (142) ceases to be valid and the relation (29) should be taken up again at the same time causing resistance R_u to become a factor as was done in the calculation preceding this comment.

Comment 2

The factor multiplying W_o in expression (136) is none other than the efficiency (P. 7.4.1.1). This efficiency could have been higher if a shorter rise time of the current had been used (Table 3). At the limit, for a negligible rise time with respect to τ, relation (89) would give:

$$W_u(\infty) = W_o \frac{1}{1 + 0,1} = W_o \times 0,91 \tag{145}$$

In order to increase efficiency still more, it would be necessary to reduce τ_2 (relation (89)) which would compel increasing the value of R_m (relation (43)).

9.1.4. Determination of the electrical circuit:

Depending on how it is desired to carry out charging and discharging of energy, as well as depending on how the discharge is to be triggered, various solutions may be found. We are going to briefly examine them. The corresponding circuit diagrams are shown on Figures 33 and 45.

First solution: /153

In our special example, the maximum available voltage of 3340 volts, since it is able to appear at the tube terminals, and as provided by relation (144), is greater than the self-priming voltage of 3000 volts of a tube which has already operated (P. 9.1.1.1.2). It is therefore possible to directly power such a tube beginning from a circuit diagram of the type shown in Figure 25. This is the case of Figure 33. The discharge electrodes of the tube are directly connected to terminals A and B of the superconducting storage coil. The calculation [97] shows, furthermore, that it is always possible to produce, at the terminals of the superconducting circuit, a voltage greater than the self-priming voltage of the tube, no matter what it may be, on condition of subsequently selecting R_m and $K\tau_2$.

The coil L can be produced using a superconducting material stabilized for example by copper or aluminum. The connections of the tube will then be directly connected to the ends of the stabilization conductor, the latter being excluded from switch K_2 [87]. The numerical values to be given to the chief components of the circuit are those calculated in P. 9.1.3.

The charging of energy into coil L will be carried out by an outside generator capable of supplying the 1540 amperes required (relation (138)). In order to carry out this charging, it will be possible to temporarily use the tube connections without having to make a disconnection, since the voltages involved (P. 3) are too low to energize it. It will be necessary, on the other hand, for these connections to be supplied so as to be able to carry the necessary current during the whole charging period (P. 4.1).

/154

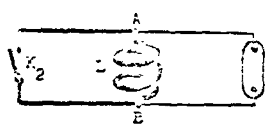


Figure 33

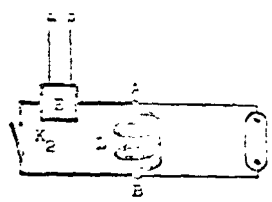


Figure 34

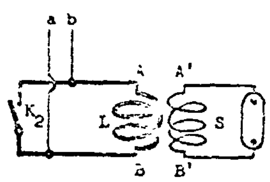


Figure 35

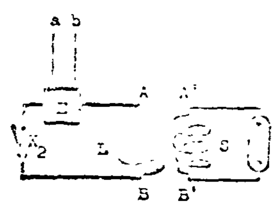


Figure 36

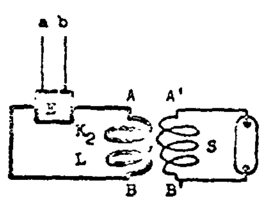


Figure 37

The superconducting parts are shown by heavy lines.

/155

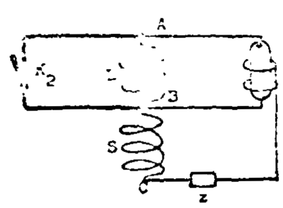


Figure 38

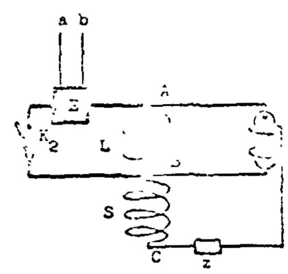


Figure 39

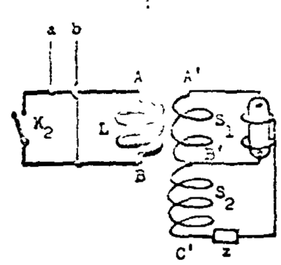


Figure 40

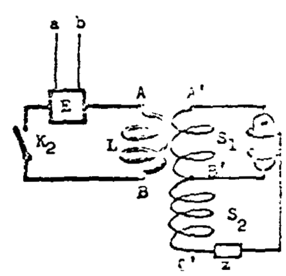


Figure 41

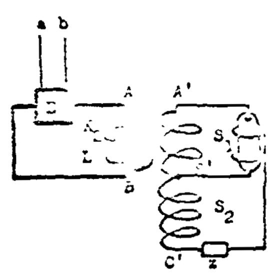


Figure 42

The superconducting parts are shown by heavy lines.

- The superconducting parts are shown by heavy lines.

- Tr designates transformer.

Figure 43

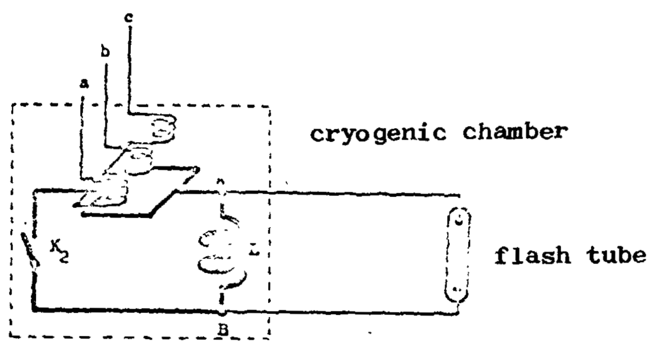


Figure 44

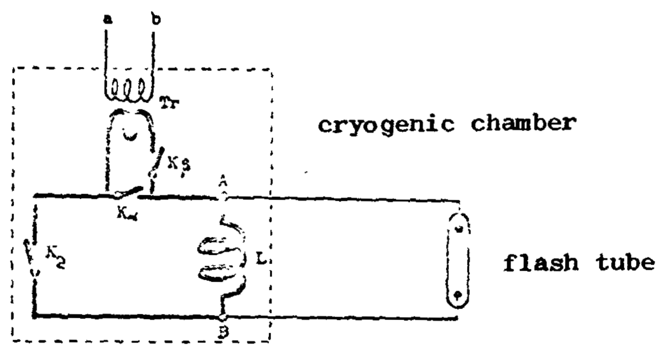
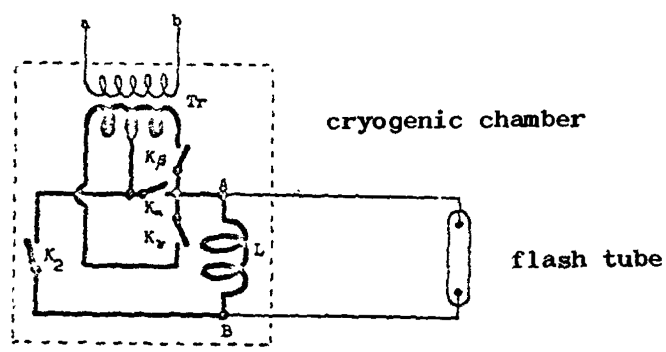


Figure 45



Second solution: /157

Depending on conditions of use, the connections required by solution 1 can introduce into the cryostat considerable energy losses. In this case, the 1540 amperes required will be produced inside of the superconducting circuit by means of an energy transformer such as shown by symbol E on Figure 34. This energy transformer will be, for example:

- a superconducting dynamo (P. 4.2);
- a flux pump (P. 4.3). Figure 43 shows the circuit powered by a static flux pump with three inductors beginning from the lead-ins of current a, b, c;
- or, if the degradation of energy [70] is to be avoided, a static transformer (P. 4.4) operating in well-determined conditions. Figure 44 shows the circuit powered by such a transformer beginning from current lead-ins a and b [70] [70a] [72] and Figure 45 shows the circuit powered by a symmetrical transformer beginning from current lead-ins a and b [70] [70a].

For operating details, see paragraphs and references provided.

The dynamo will be mechanically actuated from outside the cryostat by means of a transmission shaft which will decrease in size as power to be supplied becomes less (longer charging period) and as rotation speed increases. The flux pump or transformer can be powered by means of very low currents with voltages rising as power to be supplied increases. It is possible to have some idea of the mean power which will have to be introduced into the cryostat /158 by giving consideration to the energy to be introduced, for example: 2360 Joules (relation (137) in the case where there is no degradation of energy during charging (P. 4.4), and the maximum repetition rate: one flash every four minutes (P. 9.1.1.1.1), which gives:

$$\frac{2360 \text{ Joules}}{4 \times 60 \text{ seconds}} < 10 \text{ watts.}$$

This power can be introduced into the cryostat, for example, by means of a 10 amperes current with 1 volt, or even using a current of 44 milliamperes with 227 volts...which presents no problem.

The connections of the flash tube are only travelled by the current during period of discharge (time constant 4×10^{-3}s, Table 3). The Joule effect is, therefore, very limited in time and they do not have to be as strong as they would if they had to sustain the current during the whole charging period. At any rate, it will be advantageous to "optimize" them as a function of the temperature in their passing through the cryostat, since the electrical resistance of the conducting materials can become very low at very low temperatures [1]. For example, at 4° K the electrical resistance of pure copper can become 300 times lower than at ambient temperature and that of very pure aluminum 2000 times lower. This allows reducing, by a factor on the same order, the corresponding section of the connections.

Third group of solutions:

The superconducting circuit of energy storage can be made materially independent of the power supply circuit of the tube. The electrical coupling between these two circuits is then carried out by induction. This is the case diagrammed by Figures 35 to 37.

Since the circuit S of power supply of the tube is only traveled by the current during period of discharge, it is formed by a normal conductor. Its /159 electrical resistance should be sufficiently low with respect to the equivalent resistance of the energized tube (here 0.5 Ω) in order not to reduce the efficiency (P. 7.4.1). By keeping this circuit at low temperature, it is possible to profit under these conditions from the low resistivity of conducting materials [1] and when a material which is only slightly magnetoresistant is selected, this allows only a very small total quantity of it to be used.

The system of coupled L and S circuits does not include a magnetic circuit, since all the magnetic circuits presently known are saturated with inductions allowing superconductors.

Figure 35 shows the current introduced into the superconducting storage circuit by direct connections.

Figure 36 shows use of an energy transforming device designated by E used to charge the superconducting circuit. This allows storage of energy under a current which can be selected independent of the charging current and the current used for power supply of the tube.

In Figure 37, the switch K_2 has been left out. In reality, it is the energy storage coil which is used as switch (P. 7.5.1.3.2).

Fourth group of solutions:

When it is desired for the flash tube to be energized from the beginning of discharge from the superconducting circuit, there is the means of powering its triggering electrode using a secondary S circuit coupled by induction to the superconducting circuit. The charge of the storage current can be carried out either directly by tube connections (and this is the case of Figure 38) or through the intermediary of an energy transforming device designated by E on /160
Figure 39, of the type, for example, of those shown in detail on Figures 43 to 45.

The secondary circuit S may be very simply determined:

Let us assume that we should like to see the tube energized as soon as the voltage at its terminals reaches 1.5 kV and that in order to accomplish this, it was only needed to apply at this instant a voltage of 25 kV to the triggering electrode.

Before the discharge into the tube has begun, the system of L and S circuits (Figure 38 or 39) behaves like a vacuum autotransformer. If it is desired to produce 25 kV at the terminals of S when there are 1.5 kV at the terminals of L, it is enough to produce a transformation ratio equal to:

$$\frac{\text{vacuum voltage at terminals of S}}{\text{vacuum voltage at terminals of L}} = \frac{e_S}{e_L} = \frac{25_{kV}}{1.5_{kV}} \neq 17 \qquad (145)$$

As soon as the voltage at the terminals of superconducting coil L has reached 1.5 kV, there will then be 25 kV at the terminals of S and the discharge will begin into the tube. This discharge will be powered by the energy stored in L. When the equivalent R_u resistance in the tube is practically constant, the time constant of the discharge will have the value: $\tau = \frac{L}{R_u}$. The S circuit which will have played its role in triggering the discharge into the tube will no longer participate in the latter since it is connected to an electrode outside the tube. Furthermore, it will include, as a safety measure, in order to limit the maximum current possible, an impedance of protection z. As has been seen (P. 8.1.4.1), this impedance will be made up more particularly either by a capacity C, only allowing one pulse to pass at the beginning of discharge from the superconducting circuit, or by a resistance r. In this last case, an elementary calculation shows that in order for the maximum energy transmitted by circuit S, in case of short circuit, to be negligible with respect to the stored energy, it is enough that: /161

$$\frac{S}{r} \ll \frac{L}{R_u} \tag{146}$$

in which S designates the coefficient of self induction of the winding designated by the same symbol.

In the case just considered, the triggering of the tube is therefore controlled by the triggering of switch K_2.

Fifth group of solutions:

The superconducting energy storage circuit can be made materially independent from the triggering and power supply circuits of the tube. The electrical coupling between these circuits is then carried out by induction. This is the case diagrammed by Figures 40 to 42.

Circuit S is the analog of circuit S of Figures 35 to 37 and circuit S_2 is the analog of circuit S of Figures 38 and 39.

Figure 40 shows the current introduced into the superconducting storage circuit by various connections.

Figure 41 shows the current introduced by the intermediary of an energy transformer designated by E.

In Figure 42 the switch K_2 has been left out. Indeed, it is the energy storage coil which is used as switch (P. 7.5.1.3.2).

Just as in the preceding solutions, the determination of circuits S_1 and S_2 causes no difficulty. /162

9.1.5. Practical design:

Many possibilities are available depending on:

- the electrical circuit selected;
- the superconducting material used;
- the geometrical form of the circuit used (cylindrical, toric...);
- the "optimization" desired (P. 6.6): minimum weight, minimum space requirement, minimum cost price....

We are going to consider some of the possibilities as a function of the electrical circuit selected. In each case, the circuit will have to satisfy the conditions described in P. 7.6.

9.1.5.1. Discharges using direct connections:

9.I.5.1.1. Charging is carried out by direct connections:

This is the case with Figure 33 (P. 9.1.4, first solution) when the energization is ensured by the discharge circuit, or with Figure 38 (P. 9.1.4, fourth solution) when it is ensured by a distinct circuit.

The superconducting circuit should store energy directly under the current required for the tube, i.e. here 1540 amperes, according to P. 7.6. Its stabilization will result from the use of a material itself stabilized (P. 6.3).

In order to make the ideas more specific, we are going to provide the /163
results concerning one example, from an infinite population possible [97], determined beginning from critical characteristics (critical current as a function of the critical induction at constant temperature 4.3°K) of different materials presently on the shelf, by using the method of determining the storage coil proposed by Hassel [84].

Storage winding:

Cylindrical winding (Figure 46) forming a coil with 2×10^{-3} henry (relation (129)

- mean radius: $r = 10$ cm
- height: $h = 10$ cm
- total thickness (including insulators): $e = 2.5$ cm
- material used: stabilized cable reference

C S 8670 [111], insulated, whose characteristics and makeup of the straight section have been plotted on Figure 47.

- Number of turns of the cable: $n_L = 105$ turns (in 5 layers of 21 turns)
- induction at the center of winding: $B_o = 0.97$ tesla
- maximum induction on the wire: $B_m \neq 1$ tesla (cf. Figure 46).

Connections of the flash tube:

Let us assume that the link of the flash tube to the energy storage device requires a cable 5 meters in length. If it is desired to avoid an appreciable energy loss in this cable, it will be necessary for its total resistance R_3 to be much lower than that of the tube once energized (P. 7.4.1.1). Take, for example:

$$R_3 < \frac{R_u}{100} \tag{147}$$

from which it follows that: /166

$$R_3 < 0.005\ \Omega\ . \tag{148}$$

Since the total length of the conductors forming the cable is equal to 2 x 5 meters, this leads to a minimum section s, for each of the two conductors of the cable, equal to:

$$s = 36\ \mathrm{mm}^2. \tag{149}$$

/164

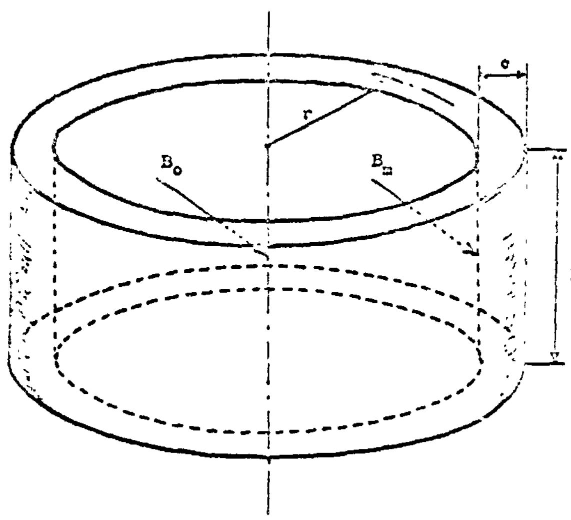
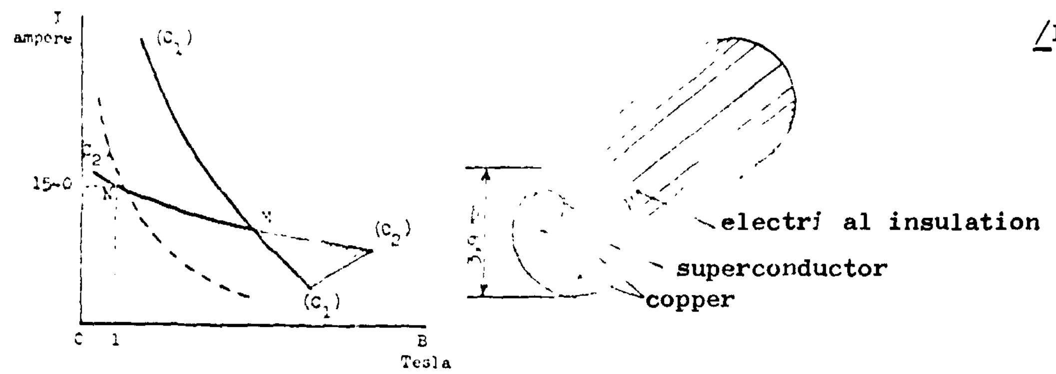


Figure 46.



/165

Figure 47. Characteristics of Cable C S 8670 (Atomics International)

Characteristic of the superconducting core alone: curve (C_1) (cf. P. 1)

- above curve (C_1) the material is in the normal state;
- below curve (C_1) and very close to the curve only a "short specimen" is superconducting. Owing to degradation, a winding is only adequately conducting below (C_1).

Characteristic of the stabilization copper alone: curve (C_2) (for a dissipation of 0.4 watt per cm^2 of outside surface):

- above curve (C_2), there is heating;
- below curve (C_2), there is an equilibrium temperature;

Characteristic of the system:

- above (C_1), there is no operation possible;
- below (C_1) and (C_2) there is neither instability nor transition (the current, which does not produce losses when it passes into the superconductor, can be forwarded by the copper when the superconductor transits).
- between both curves, in the shaded zone, the core is not superconducting on a stable basis;
- between both curves, in the unshaded zone, there is a capability for operation (however, the material only returns to the superconducting state in case of accidental transition when the current is momentarily lowered below curve (C_2)).

It is in the vicinity of M, above both curves, that the cable is best used (both insofar as concerns the superconducting core as well as the copper associated with it) since, in this case, a winding behaves like the "short specimen." The case of P. 9.1.5.1., for example, is depicted by point N: there is a poor use of the superconducting core whose section could be reduced (dotted curve in the Figure).

Switch:

- material used: Nb, Ti alloy, reference T 48 [112], with wire 0.254 mm in diameter, unsheathed copper.

- number of wires in parallel: since the total current to pass is 1540 amperes, it is appropriate to use 32 wires in parallel when it is not desired to exceed a current of 50 amperes per wire (such a wire carries from more than about a hundred amperes to several kilogauss). These wires, not insulated from each other, will be joined to form a cable.

- length of cable: in order to produce a resistance in the normal state $R_m = 5\ \Omega$ (relations (41) and (130)), there are required 13.5 meters of cable, the resistivity of the material being on the order of 60 $\mu\Omega$ x cm. This cable will be arranged so as to have a negligible coefficient of self induction L_2 with respect to L, for example:

$$L_2 < \frac{L}{100} \qquad (150)$$

i.e.

$$L_2 < 2 \times 10^{-5} \text{ henry} \qquad (151)$$

- triggering of the switch: this triggering will be carried out by a thermomagnetic process (P. 7.5.1.2). In equation (41) the value of k desired will be produced (relation (131)) by acting on the preheating temperature of the switch and on the rise rate and amplitude of the magnetic triggering pulse. /167

Triggering circuit of the tube:

This circuit, only existing in the case of Figure 38, will include n_s turns determined by the relation (valid for a vacuum transformer):

$$\frac{e_s}{e_L} = K\sqrt{\frac{S}{L}} = K\frac{n_s}{n_L} \quad K \text{ being the coupling coefficient} \qquad (152)$$

which gives n_s = 2500 turns for e_s = 25 kV, e_L = 1.5 kV, N_L = 105 turns and K = 0.7.

These turns will be formed from a very fine copper wire (for example, with a 0.05 mm diameter giving at the temperature of liquid helium a resistance on the order of 80 Ω). This represents a very low quantity of copper (at the maximum two layers on the winding at the same time taking the wire insulation into account). It is not necessary to produce with the storage circuit a good coupling of this winding which should only put into play a negligible energy and, if necessary, it is possible to easily increase the value of n_s in relation (152). On the other hand, a good insulation should be provided this winding.

The winding will be protected by a pure resistance r in series having a value, for example, according to relation (146) as follows:

$$r = 100 \frac{S}{L} R_u = 100 \frac{(2500)^2}{(105)^2} 0.5 = 28\,000\ \Omega \qquad (153)$$

When the voltage of 25 kV is reached, this resistance will therefore limit the maximum current supplied at a value on the order of an ampere. /168

When the energy thus available proves to be too low to energize discharge, this value of r would be reduced.

9.1.5.1.2. Charging is carried out by an energy converter:

This is the case with Figures 34 and 43 to 45 (P. 9.1.4, second solution) when energization is ensured by the discharge circuit or with Figure 39 (P. 9.1.4, fourth solution) when it is ensured by a distinct circuit.

As in the preceding paragraph, the superconducting circuit is to store the energy directly under the current of 1540 amperes necessary for the tube. The circuits are the same, the only difference being in the fact that it is possible to charge energy beginning from a current with different value.

9.1.5.2. Discharge using a coupled circuit:

9.1.5.2.1. Charging is carried out by direct connections:

This is the case with Figure 35 (P. 9.1.4, third group of solutions) when energization of the tube is ensured by the discharge circuit or with Figure 40 (P. 9.1.4, fifth group of solutions) when it is ensured by a distinct circuit.

Storage winding:

The superconducting circuit stores in this case the energy under a necessarily relatively low current compatible with the capabilities for current lead-ins. According to the second relation (20), the coefficient of self induction of the winding necessary becomes higher as the storage current drops. /169

Nevertheless, the total quantity of material used remains practically independent from the value of this current (p. 6.2). The winding has, therefore, practically the same dimensions as in P. 9.1.5.1.1 with almost the same space requirement for the secondary windings (cf. below).

By designating the new value of the storage current by I'_o, the required number of turns n'_L results directly from relation (21):

$$n'_L = n_L \frac{I_o}{I'_o} = 105 \frac{1540}{I'_o} \tag{154}$$

The material used will be selected as a function of the current I'_o, induction B_o at the center of the winding and the maximum induction B_m on the wire being the same as in P. 9.1.5.1.1 according to P. 6.2.

Secondary winding S:

This winding should transport the current $I_o = 1540$ amperes required for the tube during discharge. It still results from (21) that it should include a number of turns n_s equal to that of the storage winding of P. 9.1.5.1.1, i.e.:

$$n_s = 105 \text{ turns.} \tag{155}$$

The material used will be a normal conductor with low resistivity, in such a manner that the resistance R'_3 of the winding will be low with respect to that of the energized tube (p. 7.4.1.1).

If it is desired, for example, to produce the condition /170

$$R'_3 < 0.005\ \Omega \tag{156}$$

analogous to relation (148), since the total length of the conductor forming the winding is equal to 66 meters, this leads, in the case of a resistivity of $3 \times 10^{-9}\ \Omega \times \text{cm}$, to a minimum section of the conductor equal to:

$$s = 0.4\ \text{mm}^2. \tag{157}$$

This represents a very small quantity of aluminum (at the maximum one complete layer taking into account the insulation of the wire). In reality, since this winding is intended to transport all the stored energy, it will be advantageous to couple it to the maximum to the storage circuit by interweaving its turns with those of this circuit. In this case, care will have to be taken with the insulations.

Conditions of the flash tube:

Produced in the same way as in P. 9.1.5.1.1. they are connected to the terminals of secondary winding S.

Switch:

When it is produced, for example, with the same material as in P. 9.1.5.1.1 used under the same conditions (50 amperes per wire), the number n of wires to be placed in parallel is equal to:

$$n = \frac{I_d}{50} . \qquad (158)$$

In order to preserve characteristic time constant τ_2 (Table 3), the cable length to be used is that allowing achievement of a resistance in normal state R'_m confirming the relation: /171

$$\tau_2 = \frac{L'}{R'_m} . \qquad (159)$$

This relation (p. 7.5.1.3.3) ultimately leads to using the same quantity of superconductor as with P. 9.1.5.1.1. The triggering of the switch will be carried out in the same way.

Triggering circuit of the tube:

This circuit, only existing in the case of Figure 40 (winding S_2), should have exactly the same number of turns as with P. 9.1.5.1.1.

Since winding S_1 was made of aluminum, it can be preferable to do likewise for S_2 with the goal of reducing risks of corrosion in case of condensation of water vapor during rises and falls of temperature.

9.1.5.2.2. Charging is carried out by an energy converter:

This is the case of Figure 36 (P. 9.1.4, third group of solutions) when the energization of the tube is ensured by the discharge circuit, or the case of Figure 41 (P. 9.1.4, fifth group of solutions) when it is energized by a distinct circuit.

Circuits S, S_1, S_2 are identical to those of P. 9.1.5.2.1. The storage circuit can have any coefficient of self induction whatever which is compatible /172 with the charging device used. This case allows production of storage circuits with a very low self induction coefficient, storing energy under very high currents. These storage circuits can be formed by just a few superconducting turns or even a single turn [74a]. In this way, it is possible to produce circuits having an excellent stability (P. 6.3).

9.1.5.2.3. The superconducting switch is mixed:

This is the case with Figure 37 (P. 9.1.4, third group of solutions) when energization of the tube is ensured by the discharge circuit, or the case of Figure 42 (P. 9.1.4, fifth group of solutions) when it is ensured by a distinct circuit.

Circuits S, S_1, S_2 are identical to those of the preceeding paragraph.

The storage circuit can include any number of turns whatever, but it will turn out to be especially advantageous for its stability when it is formed by several turns or even by a single turn (P. 6.3). We are therefore going to consider the case of a single turn.

By using the same material and the same current density (10^5 A/cm^2) as those used by the switch in P. 9.1.5.1.1, calculation gives a thickness of e = 1.6 mm in the case of a circuit formed by a single turn with mean radius r = 10 cm and height h = 10 cm (Figure 46).

The self induction coefficient of this circuit is then: /173

$$L'' = 2 \times 10^{-7} \text{ henry} \tag{160}$$

and the current to be trapped:

$$I''_o = 154{,}000 \text{ amperes} \tag{161}$$

The resistance of this turn in the normal state is:

$$R''_n = 2.25 \times 10^{-3}\ \Omega \tag{162}$$

There would therefore be produced, when the whole turn was transited, a characteristic time constant τ_{2n} having as value:

$$\tau_{2n} = \frac{L''}{R''_n} = 0.89 \times 10^{-4}\ \text{s} \tag{163}$$

This time constant is too low when it is desired to preserve the value τ_2 given at the beginning in Table 3. The time constant τ_2 will be obtained by only causing transit of a fraction of the total circumference of the turn equal to:

$$\frac{R''_m}{R''_n} = \frac{\tau_{2n}}{\tau_2} = \frac{0{,}89 \times 10^{-4}}{4 \times 10^{-4}} = 0{,}22 \tag{164}$$

This depicts a little less than a quarter of the total circumference of the turn.

If the value of characteristic time constant τ_2 (Table 3) had not been prescribed, the fact of causing transit of the whole energy storage turn would improve the efficiency without increasing the superconducting quantity becoming /174
a factor in the circuit. More particularly, for a time t_1 of current rise, negligible with respect to τ, relation (89) would give a limit efficiency of 0.98 instead of 0.91 (relation (145)).

9.1.5.3. Cryogenic device:

In consideration of the small size of the circuit (P. 9.1.5.1.1) a conventional cryostat (P. 2) can be used.

9.1.6. Stored energy densities:

We are going to calculate respectively, in the case of P. 9.1.5.2.3, the mean energy density stored in the dielectric inside the winding as well as the energy density stored per unit of volume of superconducting material (Figure 46).

In the dielectric there is produced [54] for a mean induction of 1 tesla (P. 9.1.5.1.1):

$$w_d = 0.4 \text{ kj/liter} \qquad (165)$$

Per unit of volume of superconducting material used, there is produced an energy density:

$$w_s = \frac{W_o}{2\pi r e h} = 25 \text{kj/liter} \qquad (166)$$

Although much higher than energy densities produced with capacitors [54], these values are quite small with respect to those which can be produced with superconductors. This results from the low values of the induction and current /175
density (P. 6.4) which have been used as well as the low value of total stored energy [54].

It can be observed that, as opposed to capacitors, no circuit here is under voltage outside of the period of time during which discharge takes place.

9.2. Power supply of a high power arc: /176

The characteristics of the arc to be powered (P. 8.3.2.2) which has been selected as an example, are given in paragraph 9.2.1 and the determination of the superconducting power supply has been carried out in the following paragraphs.

9.2.1. Characteristics of the arc supplied:

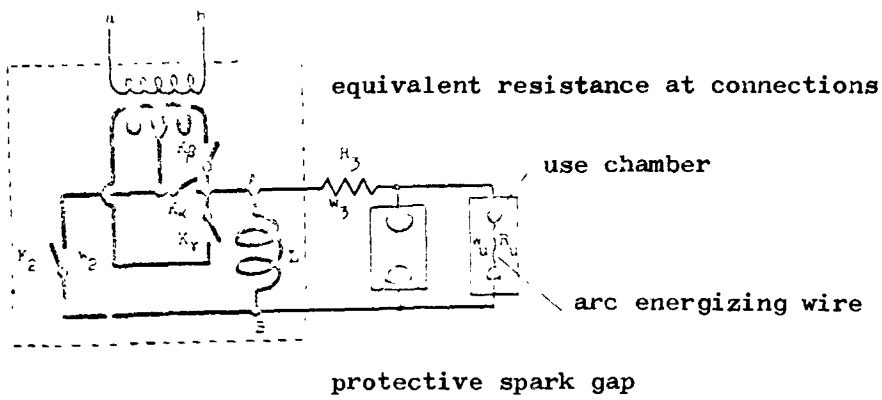
This concerns powering an arc, energized by explosion of a wire as shown in Figure 29. The characteristics of this arc are as follows:

- energy in the arc: 20 megajoules
- resistance of the arc: the arc is essentially dissipative and the order of magnitude of its resistance remains perceptibly constant during discharge and on the order of 0.03 Ω in the case of a current on the order of about a hundred thousand amperes. (Indeed, it is rather the order of magnitude of the arc voltage which stays practically constant when the current varies. Cf. the end of P. 9.1.1.3 and P. 9.3.)
- duration of arc: 0.1 second
- maximum repetition rate: 1 discharge every 4 hours.

9.2.2. Circuit diagram:

The circuit diagram is shown by Figure 48. It corresponds to the one shown in Figure 45. In the same way, the superconducting parts are shown by heavy lines. A protective spark gap has been connected in parallel on the use. It is used to limit the difference in potential which could appear at the terminals of the superconducting circuit in the case where the arc is not energized in the use chamber. This case can occur, for example, when it has been forgotten to install the energizing wire shown on Figure 48. Normally, the protective spark gap does not become a factor.

/177



Energies which become a factor: w_2 in the switch

w_3 in the connections

w_u energy used

Figure 48.

9.2.3. Initial computational data:

Since the energy involved is high, a limit will be placed on the maximum value of the losses which can be tolerated in the switch, corresponding to a calorific energy dissipated into the cryogenic device during transition, as well as on the maximum value of energy lost owing to joule effect in the connections.

Under these conditions, the operating case specified in Table 4 will be considered: /178

Table 4.

Designation of quantity	Symbol and given value	Relation to be used
Energy in the arc	$W_u = 20$ MJ	
Time constant of discharge	$\tau = 0,1$ s	(50) modified
Losses tolerated:		
in the superconducting switch	$w_2 < 0,15\ W_u$	
in the connections	$w_3 = 0,10\ W_u$	
Resistance of the arc	$R_u = 0,05\ \Omega$	

9.2.4. Determination of power supply characteristics:

The data from Table 4 allow successive determination of the following quantities:

Resistance of connections - relation (89.6)

$$R_3 = 0{,}10 \quad R_u = 0{,}003\,\Omega \tag{167}$$

Storage coil - relation (50) in which $R_u + R_3$ replaces R_u

$$L = 0{,}1\,(R_u + R_3) = 0{,}0033 \tag{168}$$

Minimum value of the efficiency

$$\begin{aligned} W_o &= W_u + W_2 + W_3 \\ &\leq W_u + 0{,}15\,W_u + 0{,}10\,W_u \end{aligned} \tag{169}$$

whence:

$$\eta = \frac{W_u}{W_o} \geqslant 0{,}80 \tag{170}$$

Minimum value of R_m - relation (90) /179

$$\frac{R_u\ R_m}{(R_u + R_3)(R_u + R_3 + R_m)} \geqslant 0{,}80$$

whence:

$$R_m \geqslant 0{,}242\,\Omega \tag{171}$$

In the case of $R_m = 0.242\Omega$, the value of the efficiency $\eta = 0.80$ is only strictly obtained when the transition velocity of the switch is infinite (P 7.4.1.1). Indeed, we have seen (P. 7.4.1.1, expression (90.1)) that this limit value of the efficiency was practically reached in the case where b' was small with respect to unity. In order to preserve more capabilities in the following, we are going to use for R_m a value greater than 0.242Ω, for example:

$$R_m = 0{,}3\,\Omega \tag{172}$$

Under these conditions, we are going to calculate the exact value of the efficiency.

Exact value of the efficiency

Under these conditions:

- the exact value of the efficiency is, according to relation (90):

$$\eta_{max} = \frac{0{,}03 \times 0{,}3}{0{,}033 \times 0{,}333} = 0{,}82 \tag{173}$$

- the exact value of the efficiency is given by relation (90.1) in which /180
there is:

$$a' = \frac{R_m + R_u + R_3}{R_m} = \frac{0{,}334}{0{,}3} = 1{,}11 \tag{174}$$

$$b' = \frac{2\,(R_u + R_3)}{kL} = \frac{2 \times 0{,}033}{10^3 \times 0{,}0033} = 0{,}02 \tag{175}$$

for

$$k = 10^3_{\ s^{-1}} \text{ (value to be used by us later on)} \tag{176}$$

whence:

$$\alpha' = b' \frac{a'-1}{a'} - 2 = 0{,}00198 - 2 \approx -2 \tag{177}$$

(The error introduced into the calculation by this approximation is totally negligible)

$$\beta' = \frac{b'}{a'} - 1 = -0{,}972 \tag{178}$$

The relation (90.1) gives in this case:

$$\eta = \frac{0{,}03}{0{,}033} \times 0{,}02 \times (0{,}11)^{-0{,}00198} \times (44{,}00 + 0{,}47 + 0{,}16 + 0{,}06 + \ldots) = 0{,}818 \tag{179}$$

By comparing this value to the maximum limit given by relation (173), it can be seen that the value of k selected with (176) is sufficiently high and that practically nothing would be gained by increasing it further.

To this value of k correspond the values of t_1 and I_m calculated in the following.

Initial energy to be stored /181

$$W_o = \frac{W_u}{\eta} = \frac{2 \times 10^7}{0{,}818} = 2{,}45 \times 10^7 \text{ joules} \tag{180}$$

Storage current

$$I_o = \sqrt{\frac{2W_o}{L}} = 122{,}000 \text{ amperes} \tag{181}$$

Maximum value of the discharge current

This maximum value is given by the relation (62.9) at instant t_1 specified by relation (44). We are therefore going to calculate t_1. In order to do this, we must calculate according to (43):

$$\tau_2 = \frac{L}{R_m} = 1{,}1 \times 10^{-2}\, s \tag{182}$$

whence it follows, according to (44):

$$t_1 = 2{,}58 \times 10^{-3}\,s \tag{183}$$

and according to (55):

$$y_1 = 0{,}0774 \tag{184}$$

The relation (62.9) gives in this case:

$$I_m = 122.000 \times (0{,}11)^{-0{,}00198} \times (1-0{,}0774) \times (1{,}11-0{,}0774)^{0{,}00198-1} \times (0{,}0774)^{-0{,}009}$$

$$\approx 122.000 \times \frac{0{,}923}{1{,}033} \times 1{,}022 \tag{185}$$

$$= 112.000 \text{ amperes}$$

By way of confirmation, it is now possible to perform the following calculation:

Energy dissipated into the switch /182

The relation (62.6), in the different terms of which $R_u + R_3$ should replace R_u, gives:

$$w_2(\infty) = 24{,}5 \times \frac{0{,}033}{0{,}3} \times 0{,}02 \times (0{,}11)^{-0{,}00198} \times (45{,}10+0{,}71+0{,}33+0{,}20+0{,}13+0{,}09+\ldots)$$

$$= 24{,}5 \times \frac{0{,}033}{0{,}3} \times 0{,}02 \times 1{,}004 \times 46{,}6 \qquad = 2{,}5 \text{ Megajoules} \tag{186}$$

Available energy

From relation (89.7) and (169) it follows that:

$$w_u = (w_o - w_2)\ \frac{R_u}{R_u + R_3} \tag{187}$$

whence:

$$w_u = (24{,}5 - 2{,}5)\ \frac{0{,}03}{0{,}033} = 20 \text{ Megajoules} \tag{188}$$

Energy dissipated into the connections

The relation (89.6) gives directly:

$$w_3 = 20\ \frac{0{,}003}{0{,}03} = 2 \text{ Megajoules} \tag{189}$$

The characteristics of the power supply specified in this way are compiled /183
in Table 5.

Table 5.

	Resistance	Coil	Time constant	Maximum current	Energy involved
Use circuit	$R_u = 0{,}03\,\Omega$	0	$\tau = 0{,}1\,s$	$I_m = 112.000\,A$ ($t_1 = 2{,}58 \times 10^{-3}\,s$)	$w_u = 20$ MJ
Storage circuit	0	$0{,}0033\,H$	∞	$I_o = 122.000\,A$	$w_o = 24{,}5$ MJ
Switch	$R_m = 0{,}3\,\Omega$ ($\alpha = 10^3\,s^{-1}$)	0	$\tau_2 = 1{,}1 \times 10^{-2}\,s$	$I_o = 122.000\,A$	$w_2 = 2{,}5$ MJ
Connections	$R_3 = 0{,}003\,\Omega$	0		$I_m = 112.000\,A$	$w_3 = 2$ MJ

Efficiency: $\eta = \frac{w_u}{w_o} = 0{,}82$

9.2.5. Practical design:

Just as in paragraph 9.1.5, many design possibilities are available. We are only going to give one example of them. The calculation of the storage winding was carried out according to the method already **described** (P. 9.1.5.3.1), as proposed by Hasseli [84] (cf. Figure 46).

Storage winding

mean radius: $r = 75$ cm

height: $h = 75$ cm

total thickness: $e = 15$ cm

material used: Nb-0, 48 Ti alloy in the form of wires embedded in a copper matrix of very high purity

number of turns: $n_L = 49$ turns

current density in the superconductor: $\delta = 10^5$ A/cm^2 /183

useful section of the superconducting alloy: 1.22 cm^2

total weight of superconducting alloy: 160 kg

total weight of the copper of stabilization: 3000 kg

induction in the center of winding: $B_o = 4.4$ tesla

maximum induction with the superconductor: $B_m < 5$ tesla

Connections

The outside part of the cryostat will be designed, for example, with a very thick copper section. The lead-ins into the cryostat will be made of copper with very high purity, allowing their resistivity to be lowered by a factor on the order of 200 at 4°K. These lead-ins will be used at the same time as a piping system allowing recovery of gaseous helium coming from vaporization of the liquid helium contained in the cryostat and will be considerably cooled in this way. The effective cross section of the copper will progressively decrease towards their ends located in the very low temperature region where they will be directly connected to the ends of the copper winding used for stabilization of the energy storage winding [87].

The copper sections used to produce these connections will be selected so as to:

- obtain a total resistance $R_3 = 0.003\ \Omega$ (Table 5);
- dissipate during discharge, into any section of the circuit whatever

formed by these connections, only an energy less than the energy which would cause a rise in temperature dangerous for the insulators used. By designating by r the resistance in ohms of any section whatever of the circuit forming the connections, the energy w expressed in joules dissipated during discharge is given under conditions considered by the direct relation: /185

$$w_{joule} = \frac{r}{0{,}003\,\Omega}\, 2 \times 10^{[illegible]} \qquad (190)$$

(for this dissipated energy is proportional to the resistance and $w = w_3$ should be had for $r = R_3$ (Table 5)).

Switch

It will be of the transition type (P. 7.5.1), formed beginning from a same superconducting alloy as the storage winding but electrically insulated and thermically stabilized, for example, by means of a metallic oxide with high thermal conductivity.

When the switch is produced without coil (P. 7.5) and when the precaution is taken not to arrange it in the field of the storage circuit, the magnetic field in the vicinity of the superconductor which forms it can remain very small (for example, several tenths of a tesla at the maximum). Under these conditions, the critical current of the superconducting material will be higher [112] and the switch can be produced at the same time preserving the same effective superconducting cross section as in the storage winding, i.e. 1.22 cm^2.

It follows from this:

- the length l of the superconductor to be used (relation (99)) knowing that the resistivity of the material after transition is 70 μΩ x cm [113]:

$$l = \frac{0{,}3 \times 1{,}22}{70 \times 10^{-6}} = 5250 \text{ cm} \qquad (191)$$

- the volume sl of superconductor to be used: /186

$$sl = 5250 \times 1{,}22 = 6400 \text{ cm}^3 \qquad (192)$$

i.e. 6.4 dm^3; this result also being deduced from (101);

- the energy dissipated during discharge, per unit of volume of superconductor:

$$\frac{w_2}{sl} = 390 \text{ joules / cm}^3 \qquad (193)$$

i.e. 93 calories/cm^3 which approximately corresponds to 15 calories per gram of superconductor.

The order of magnitude of corresponding maximum rise in temperature will remain very small compared, for example, to the value of the ambient temperature.

In order to limit the quantity of helium vaporized the switch will be installed in a cryogenic compartment containing only a small quantity of liquid. This liquid helium, furthermore, can be removed beforehand by injection of gas under pressure.

Energy charging device

The charge will be carried out by means of a converter as shown on Figure 48. The duration of charge foreseen will be on the order of 20 minutes (commutations not included).

The switches K_α, K_β, K_γ, still of the transition type, will be formed from the same material as the preceeding switch. Their characteristics are determined in this case as follows:

rate of variation of charging current:

$$\frac{I}{t} = \frac{122.000_A}{20 \times 60_A} = 100_{A/s} \tag{194}$$

maximum voltage at the terminals of the storage coil: /187

$$V = L \frac{dI}{dt} = 0.0033 \times 100 = 0,33 \text{ volts} \tag{195}$$

In each one of the switches there is tolerated a total dissipation of energy during the charge equal to 10^5 joules (low value with respect to the 2.5×10^6 joules dissipated in switch k_2 during discharge). Under these conditions, the following quantities may thence be deduced:

normal resistance R_m of the switch:

$$R_m = \frac{(0,33)^2}{10^5} 20 \times 60 = 1,3 \times 10^{-3} \Omega \tag{196}$$

corresponding current into the normal switch:

$$I = \frac{V}{R_m} = \frac{0,33}{1,3 \times 10^{-3}} = 230 \text{ amperes} \tag{197}$$

power dissipated into the normal switch:

$$P = \frac{10^5}{20 \times 60} < 84 \text{ watts} \tag{198}$$

(it should be noted that this power is no longer dissipated into the liquid helium beginning from the instant at which the switch is no longer in contact with this helium).

Length of the superconductor (the section remains 1.22 cm^2):

$$l = \frac{1.3 \times 10^3 \times 1.22}{70 \times 10^{-6}} = 23 \text{ cm} \qquad (199)$$

<u>Cryogenic device</u>

The cryostat will be constructed around the energy storage winding as shown in Figure 49. It will therefore have the shape of a torus. The thermal screens which it will include inside will be interrupted in such a manner as not to form a closed turn coupled by induction to the storage winding. The cryostat itself will be formed by the stainless steel sheet with high electrical resistivity in such a manner as to reduce the joule effect resulting from the currents induced during discharge. /188

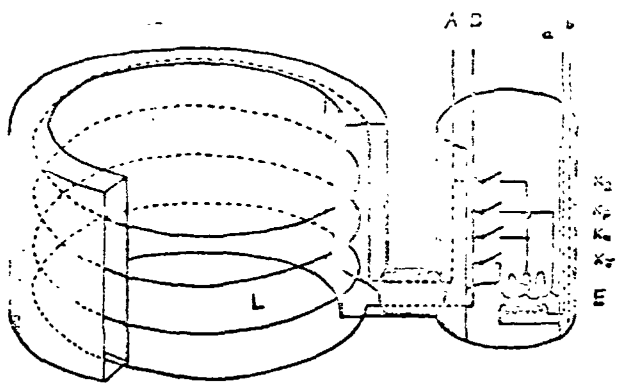


Figure 49.

It is possible to compute an upper limit of the maximum electromotive force $\mathcal{E}_m$ induced into the cryostat during discharge: knowing that the storage winding includes 49 turns and that the maximum electromotive force of which it is the seat during discharge is equal to $R_u I_m$ = 0.03 x 112,000 = 3360 volts, the maximum electromotive force developed in the cryostat is less (poor coupling of the cryostat to the storage winding) than that developed in a winding turn, it will follow that: /189

$$\mathcal{E}_m < \frac{3360}{49} < 70 \text{ volts} \qquad (200)$$

Since the electrical resistance of the closed circuit formed by the sryostat is known, there may easily be deduced an upper limit of the corresponding energy dissipated during discharge.

The energy charging converter as well as its switches K_2, K_α, K_β, K_γ will be located in a second cryostat connected to the previous one by its cold part and located close by in a region of the space subjected to a low value of the magnetic field, as indicated by Figure 49.

The cooling of the system of the device, its maintenance in the cold state, thermal control of the switches, then upon conclusion of the experiments the rise in temperature will be ensured by a refrigerator operating in closed circuit with the whole system.

Density of stored available energy /190

The density of stored available energy per unit of volume of superconducting material may be deduced directly from Table 4 and relation (192):

$$\frac{W_u}{sl} = \frac{2 \times 10^7}{6400} = 3100 \text{ Joules}/_{cm^3} \qquad (201)$$

This density of stored available energy is greater than that possible as a product of today's chemical explosives. Now, this application of superconductors is still quite new. The superconductors themselves are still in the development stage and the obtainable energy density rises with increase in capacity of the storage device [54].

Available power

During discharge, the power available is on the order of:

$$\frac{W_u}{\Delta t} = \frac{20 \text{ MJ}}{0{,}1_s} = 200 \text{ Megawatts} \qquad (202)$$

This is the order of magnitude of power supplied on the grid of an electrical power station of Electricité de France.

9.3. Comments:

When the following conditions are produced:

- arc voltages V_{arc} practically constant during discharge (beginning from instant $t_{energizing}$ at which the arc is energized);
- impedance $R_u(t)$ of the arc essentially dissipative (and this time, consequently, as a function of time); /191

- negligible impedance of connections;

-K_2 switch not of coil nature;

n obtains the following results (Figure 50):

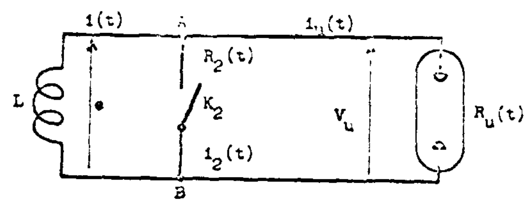


Figure 50.

Before energizing the arc

Everything occurs as if the use circuit was not connected and there appears between terminals A and B of the storage circuit a difference in potential given by relation (29) in which it is necessary to replace resistance $R(t)$ by the single resistance $R_2(t)$ then in the circuit. (When voltage $V_{energizing}$ of energization of the arc is reached, the arc will be energized. The value $i_u(t_{energizing})$ of the current in the storage coil will then be given by relation (28) in which $R(t)$ has already been replaced by $R_2(t)$ and where t will be replaced by $t_{energizing}$).

After energizing the arc /192

The arc voltage will be specified by hypothesis at constant value V_{arc} (less than $V_{energizing}$) and the Kirchoff equations applied to the circuit of Figure 50 will allow writing:

$$i(t) = i_u(t) + i_2(t) \qquad (203)$$

$$- L \frac{di(t)}{dt} = R_2(t)\, i_2(t) = R_u(t)\, i_u(t) = V_{arc} \qquad (204)$$

From relations (204) it follows that:

$$i(t) = i(t_{energizing}) - \frac{V_{arc}}{L} [t - t_{energizing}] \qquad (205)$$

and by taking relation (203) into account the current in the arc is obtained:

$$i_u(t) = i(t_{energizing}) - \frac{V_{arc}}{L} [t - t_{energizing}] - \frac{V_{arc}}{R_2(t)} \qquad (206)$$

In the case where, for $t \geq t_{energizing}$, the resistance $R_2(t)$ of the switch is high enough for the last term of equation (206) to be negligible. The current $i_u(t)$ in the arc is a decreasing linear function of time which can be diagrammed by Figure 51.

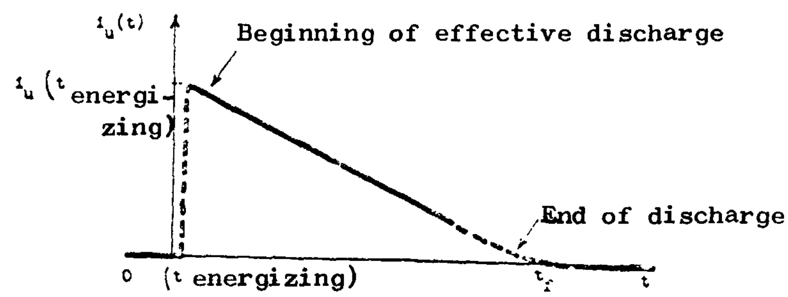


Figure 51.

Under these conditions, the last relation (204) allows calculation of function $R_u(t)$: /193

$$R_u(t) = \frac{V_{arc}}{i_u(t)} \tag{207}$$

in which current $i_u(t)$ is given by relation (206). The result is that when voltage V_{arc} is constant, the resistance $R_u(t)$ of the arc is necessarily variable and increases with time. (This result can be reconciled with the experimental curves compiled in Figure 32.)

Then, at the end of discharge, the arc will progressively be extinguished (dotted line on Figure 51).

It is possible to calculate:

$$W_u = V_{arc} \int_0^{\infty} i_u(t)\, dt \tag{208}$$

the current $i_u(t)$ being given by relation (206) and remaining zero after instant t_f in which it is reduced to zero in relation (206).

The energy dissipated in the switch:

$$W_2 = V_{arc}^2 \int_0^{t_f} \frac{dt}{R_2(t)} \tag{209}$$

It is then possible to calculate the efficiency of energy transfer by proceeding in the same way as in P. 7. We shall provide no further details.

It should finally be noted that the fact of considering the arc voltage as constant during discharge only forms an approximation.

Comment 2 /194

The two examples of designs which have just been considered in paragraphs 9.1 and 9.2 make specific today a revolutionary and forward looking application of superconductors. Nevertheless, both these examples are based on present-day materials just as they are found today. As far as examples are concerned, they are condemned to serve for the next few years as horrible predecessors until these present-day materials are replaced by well-designed superconductors which are well suited to our problem.

10. Comments Relating to the Superconducting Material and Conclusion /196

It is the prospect of using superconductors for production of intense magnetic fields which has since 1960 provided the stimulus to research and development efforts. These developments are reflected by:

- the appearance on the market of some materials such as the alloys Nb-Zr, Nb-Ti, the compound $SnNb_3$, presently processed in the form of wires, cables, strips...;
- the removal on a preliminary basis of superconducting materials which could not be produced on a practical basis in these forms;
- the obtaining in the case of processed materials, of as high as possible maximum critical inductions, to the point where it now appears difficult to see in the near future any increase in their critical inductions by large orders of magnitude (thus, it is still not known how, for example, it would be possible to multiply the critical induction of one of these materials by 10, an induction which is already quite close to the limits foreseen by actual theory. Nevertheless, combinations between different materials [85] are not to be excluded);
- the development of effective techniques of stabilization of the material as well as of the winding.

However, on the other hand, it is possible to ascertain that the critical currents produced, for example, with materials of the same nature, but with different origins, or even used with the same inductions but under different experimental conditions, sometimes differ by a factor much greater than 10. This should not be surprising when it is noted that the basic characteristic of the material, for production of intense magnetic fields, is to possess the highest possible critical induction. On the other hand, we have seen that for storage and discharge of electrical energy, it is the critical current which chiefly becomes a factor.

Indeed, when the critical current density δ_c of a given material is multiplied by 10 only, the result is that: /197

- according to relation (24), multiplication by $(10)^{3/2}$ is performed, i.e. 31.6, the stored energy, whereas if it were desired to multiply critical induction B_c by 10 (which furthermore is theoretically not possible at the present time) multiplication would only be performed by $(10)^{1/2}$, i.e. 3.16, the stored energy;

- according to relation (102) it would be possible to multiply the resistance R_m of the switch by 10 for the same mass of superconducting material used, or even to divide the mass of the switch by 10 for a same value of resistance R_m.

Now, when it is observed that the orders of magnitudes of current densities presently obtained experimentally are much less than the orders of magnitudes of the theoretical maximum possible current densities [113], an improvement of the critical current densities tending to bring them nearer to this limits could have revolutionary consequences for our application.

Whether this may be, for example, to produce maximum values of the product $\delta_c \sqrt[3]{B_c}$ (relation (24)) or of the product $\rho \delta_c^2$ (relation (102)) or just a compromise between these conditions, it does not appear that research has as yet been undertaken either on the nature of materials to be used or on the processing to be given them to put them in shape.

As far as the nature of the materials is concerned, a systematic exploratory study remains to be carried out on all superconductors known in order to determine those which can be used advantageously to form all or part of the storage and energy discharge circuits. Likewise, the search for new materials remains to be undertaken in this direction.

Insofar as the structure and shape of the materials is concerned, research remains to be done on thin layers, for example, or on massive circuits having an anisotropy (superconductivity in one direction and not in both perpendicular /198 directions), such that the trapped lines of current are "frozen" into the material and into the circuit. Our ideas along these lines appear to be like those of Abrikosov [10] who foresees a more certain future for layers with thicknesses of from 10^{-5} to 10^{-6} than for wires presently used in windings.

Likewise, a study of special properties, for example the advantageous property possessed by some materials of having higher critical values for inductions greater than a specific value [114] [85], remains to be done.

The possibility of using massive circuits (whether anisotropic or not) or circuits in layers, etc...in short circuit on themselves never appears to have been considered. Except in the production of cryotrons, the use of thin layers to obtain high critical current densities, i.e. in consequence of the high resistances in the normal state, does not appear to be in vogue. On the contrary, the use of superconductors to create magnetic fields had led in the reverse of condition (102), to development of materials having as low as possible equivalent electrical resistivities. Likewise, it was sought, in a winding, to obtain the reverse of...high-speed, homogenous and total transitions! The possibility of producing circuits formed by layers closed on themselves and in this way having a good stability in the superconducting state, a low thermal inertia and an easy access for a homogenous penetration of the magnetic induction during transition, finally a relatively high resistivity in the normal state, does not appear to have been used.

Perhaps all this has not yet been considered? It was certainly necessary, in order to have the opportunities of arriving at an advantageous result, to use the imagination to develop solutions quite different from the conventional solutions to which we have been so long accustomed. Perhaps this has been thought of? However, it is not yet very well understood how to introduce energy into such circuits on a practical basis, nor how to take energy from them. The technological problems to be solved were still too many in number /199
and their nature too unusual....

Hopefully, the spin-off from all this will show that the optimum characteristics required of material for storage and discharge of electrical energy are not the same as those required of it for producing intense magnetic fields. On the other hand, there are some points in common: for example, the requirement for producing a stable current and, in the case of operating with high induction, the requirement for obtaining from the circuit (not necessarily from the material) a sufficiently high mechanical resistance so that it can without danger resist magnetic pressure.

Given the dispersion existing, in the case of presently available materials, in the values of the characteristics most important for our application and being given that up until now no known study has been made to orient these characteristics toward values advantageous to us, it can be considered that it is now possible, having defined the problem, to produce great improvements in a short time and at low cost.

Remaining quite realistic about it, we have grounds for expecting this today.

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